

STUDENT MOTIVATION IN UNDERGRADUATE CALCULUS: FIRST THOUGHTS

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ABSTRACT. What is it that really motivates students to study mathematics, and how can this enthusiasm be cultivated in the calculus classroom? After a bit of theory as a backdrop, the experiences of the author in his first year as a graduate student instructor (GSI) for second-semester calculus are chronicled, followed by some thoughts and suggestions for other GSIs in mathematics.

Terrel Bell, former Secretary of Education, has this to say:

There are three things to remember about education. The first is motivation. The second one is motivation. The third one is motivation. [C2, 171]

But aren't undergraduate students supposed to be self-motivated? After all, why would they seek out a college degree in the first place unless they wanted to learn? Quite to the contrary, student motivation—even at the university level—cannot be presupposed or taken for granted. The consequences of an unenthusiastic classroom, as Terrel Bell points out, can be deleterious. In many ways, motivation is a critical undercurrent in the learning environment and can be a crucial determinant of student achievement.

However, students themselves are not often accustomed to thinking about their own motivation and goals. Many calculus students, for example, are first-year students and are used to the low standards of high school; they did not have to do much in order to “get by” then and may not know how to meet real mathematical challenge. Worse still, even after acclimating to the college environment, many students are only driven by the nettlesome desire to get a good grade, especially those taking the course as a requirement for an impacted major (such as bioengineering or computer science). Research indicates that this “extrinsic” motivation often comes at the expense of real comprehension and learning and can in the long run contribute to a deterioration of the teacher/student relationship as it becomes mediated by an extended calculation of partial credit on homework and exams.

Therefore, perhaps it is incumbent upon instructors to consider some questions of student motivation. What is it that really (“intrinsically”) motivates students, and how can this enthusiasm be cultivated in the calculus classroom? After a bit of theory as a backdrop, the experiences of the author in his first year as a graduate student instructor (GSI) for second-semester calculus are chronicled, followed by some thoughts and suggestions for other GSIs in mathematics.

1. A BIT OF THEORY

In the research literature, student motivation has been studied a great deal. One theory relates motivation to self-worth: “achievement goals adopted by students ... reflect a Promethean, life-spanning struggle to establish and maintain a sense of worth and belonging in a society that values competency and doing well. In effect, in our society individuals are widely considered to be only as worthy as their ability to achieve” [C2, 181]. Consider students, for example, who withhold effort or otherwise handicap themselves so as to avoid the debilitating consequences of failure—at the end of the day, they can blame not trying or procrastination rather than themselves if they do not succeed. Consider also the overstrivers, those whose manic need to achieve goes unreasonably far so as to escape any feeling of incompetency.

Motivation can either be intrinsic and extrinsic (though sometimes these are related in a complementary fashion). An externally motivated student may study to maintain a high GPA or to please his or her father, for example—the key controlling factor is external reward (or punishment). This is exhibited in the following familiar scenario (recalled by Covington):

Teacher: What did you get out of the class?

Student: I got an A.

Intrinsically motivated students, rather, follow an innate curiosity: “learning becomes valued for what it can do to benefit the individual or enhance one’s effectiveness” [C, 20]. On the whole, intrinsic motivation is positively correlated with creativity, problem-solving capacity, and deep comprehension: “Intrinsically motivated individuals have shown better qualitative performance than individuals motivated by external rewards or constraints” [SM, 266]. As Sansone and Morgan point out, however, multiple goals may be relevant to an interest in a (mathematical) task, and this multiplicity can complicate the study of student motivation.

2. THREE TESTS

In order to assess the relevance of self-worth theory and intrinsic motivation in the calculus classroom, three classroom experiments were conducted. These assessments are meant to be mainly anecdotal and suggestive.

Students were surveyed in the first week of class and asked to identify their personal goals in taking the course. Many answered this by saying that the class was required for their major or a prerequisite for another class, others felt they were there to “get an ‘A’”, or worse, to “get a passing grade and fulfill a requirement”. Only a minority listed having fun or understanding as a personal goal! In fact, most responded that, essentially, the purpose of the discussion section was to answer questions from the lecture, to help them on their homework and to prepare them for the exams.

Discouraged by these answers, a second test was administered a month later. On a weekly quiz, the students were asked to identify an aspect of mathematics that they found pleasurable and to briefly describe the last time they had felt this way about the subject. The two most common responses to this question are perhaps unsurprising: there is joy to be found in solving a difficult problem and pleasure inherent to the certainty of deductive mathematics. What was a bit surprising, perhaps, is how many students recalled recent experiences in the calculus class, quite in contradiction to the uninspired survey responses given at the beginning of

the semester! In fact, a handful of students expressed delight in solving the first problem on the quiz, which concerned the evaluation of the integral

$$\int \frac{x}{\sqrt{x^4 - 9}} dx,$$

which could be solved by trigonometric substitution but only after letting $u = x^2$. The discovery of what was required to solve the problem, a synthesis of two strategies, allowed (some) students to feel a personal sense of accomplishment and competency. In fact, many of them even commented that repetitive exercises could not garner the same sense of accomplishment as the more challenging problems and that they were most satisfied that the quiz was not trivial! Therefore, anecdotally at least, students already share in the apprehension of mathematical beauty—and it should be easy for the instructor, who has chosen to pursue mathematical study as a career, to convey this delicious sense of enchantment.

In the third experiment, students were given an optional “challenge problem” in which they would receive extra credit on their weekly quiz (allowing them to complete the task in an unhurried environment). They were briefly introduced to the idea of intrinsic and extrinsic motivation; they were then asked to write out and submit in a week what motivated them *intrinsically* to study mathematics and what they might do practically to put this to use before the second midterm. The results were remarkable. Students described the desire to enhance their knowledge of the subject and the world; many noted the striking way in which mathematics can explain the teeming randomness of the world in a precise and predictable way.

Many more students described a sense of pride and satisfaction in mastery and competence, especially when taking a seemingly complicated problem and getting a simple, correct answer. One response illustrates this answer (and the hidden role of self-worth) rather pointedly:

Math is sometimes hard, and sometimes easy. When it’s easy, I mean, when I understand the material when it’s first given to me, I feel smart as an owl, sharp as a pencil, and quick as a fox—you know what I mean. But when I don’t grasp new material as quickly, or when I don’t understand a problem when I first look at it, I admit that I do get discouraged at first. I’ll eat some chocolate or something to alleviate the minor depression, and then take a stab at it again. And when I study the new material again on my own and understand it, or when I solve the difficult problem, the feeling is so much better than when math is easy. I feel so proud of myself when this happens, and it’s just so gratifying. That’s why I study math.

Another puts it even more directly: “I strive to do well in math for pride. I get extremely excited when I do well in a math class or when I get correct answers. On the other hand, I take it as a personal belittlement when I cannot get a correct answer.”

On the whole, though, students were not able to enumerate specific ways in which they could apply this to studying for the midterm. Most were very generic, such as doing many additional practice problems.

Since this assignment was optional, undoubtedly the students that chose to submit a response (totalling about half of the class) were those who were already

self-motivated, hence the applicability of these responses is somewhat limited. The commonality of the themes addressed by the respondents does suggest, however, that something more is something going on here—something that can be used as a vehicle to cultivate enthusiasm in the calculus classroom.

3. SOME PRACTICAL RECOMMENDATIONS

In parallel to the research which discusses theories of motivation, there has been a fair amount of study on the practical end as well, for example, by Forsyth and McMillan [FM]. Note that many of these suggestions will save the GSI time, rather than requiring extra effort. No doubt this list is far from exhaustive.

Emphasize mastery. Since students seem to respond strongly to the sense of accomplishment in getting the right answer, GSIs can utilize this intrinsic motivator by emphasizing to their students that their goal should be to master the subject material, rather than competing with others or grades. Encourage students to associate failure with factors that can be controlled—in this way, they can avoid the debilitating consequences of failure which lead to self-handicapping. When faced with a grade-focused student, one might emphasize how much grades change over time and the limited role of the student's GPA in the bigger picture.

An important part of this suggestion is to help students set goals. If the above anecdotal evidence is any indication, it is unlikely that students on their own enumerate realistic course goals. Help students develop practical strategies for achieving these goals, thereby counteracting the tendency of even the brightest students to handicap themselves through procrastination or self-induced anxiety.

Be enthusiastic. Students reflect the enthusiasm of their instructor. Studies show that students learn best when they perceive that the GSI believes in the goals of doing well (and has set up infrastructure for the student) and believes in them not just as students but as persons. There is no substitute for the instructor that cares. The GSI can exhibit this charged concern by introducing the course and each topic in an interesting way. After all, the study of mathematics is filled with a sense of exploration, mystery, perfection—even stupefaction. Recall how you felt the first time you heard:

- $\sum_{n=1}^{\infty} 1/2^n = 1$ (Zeno's paradox),
- that there are infinite horns with finite volume,
- the amazing relationship between the functions e^x , $\cos x$, and $\sin x$ in the context of complex numbers,
- that $\infty - \infty$ can be any real number, or
- $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$.

Many of these topics are related exactly to material in a second-semester calculus course! Do the students even get a chance to experience the awesome beauty of these ideas for themselves? Can they see beyond the surface difficulty to the simplicity underneath? As Krantz puts it, "It is something of an oversimplification, but still true, that a portion of the teacher's role is as a cheerleader. You are, by example, trying to persuade the students that this ostensibly difficult material is doable" [K, 37]. Enthusiasm is listed in *The Quest for Excellence in University Teaching* as the most frequently cited attribute of excellent teachers, whether one asks students, colleagues, or administrators [B, 173]. Enthusiasm is infectious and should be a top priority of GSIs in their sections.

Model problem-solving strategy. The purpose of the discussion section is not just to echo the lecture or help the students with their homework. Especially for the fresh new faces in the calculus class, GSIs act as models of problem-solving strategy and ethics. How many students honestly think about which strategies are applicable to a particular problem before they dive in? When faced with a new or novel problem, however, they quickly find that they cannot rely on formulae or the obvious attack; they must learn how to apply what they know to the problem at hand, and often invoke a bit of creativity. Here, many students are faced with the terror of the unknown (often especially compounded in an exam setting), and even if they know how to swim in a lap pool, they may not yet be capable of braving the choppy seas on their own. Here, self-worth theory indicates that most students will avoid at great expense investing personally in solving a problem, lest they are unable to figure it out and thereby feel the sting of incompetency.

In order to overcome this barrier to motivation, it is imperative that GSIs talk aloud about the ways in which they approach problems and why a particular strategy is relevant—acting as a lifeguard of sorts, ready nearby with a kickboard. GSIs should model the thinking and reasoning process which lay behind the scenes. In communicating some general principles and strategies (solve a simpler problem, relate the problem to a similar one, work examples, and so on), the student is learning not just mathematics but how to think about thinking about mathematics. As I.M. Gelfand puts it, “The most important thing a student can get from the study of mathematics is the attainment of a higher intellectual level.” This requires a certain amount of forbearance, because students will want their GSI to do their homework for them. GSIs should learn how to ask good questions to lead the students themselves to a solution, acting as a referee in arguments about the reasonableness of students’ suggestions rather than as an answer book.

Provide context. Especially at the level of first-year calculus, proof is not a top-down, definition-proposition-theorem deductive procedure—proof is persuasion. “[S]tudents (and others, too!) do not generally learn *axiomatically* (from the top down). In many instances it is more natural for them to learn *inductively* (from the bottom up)” [K, xii]. In other words, motivating the material by providing context is a great way to motivate students to investigate it for themselves.

Consider the following example [A]: What is a group, after all? Some would say that it is a set with a binary operation under a set of four axioms, all easily forgettable. The history bears out differently, as groups were initially only permutation groups, as transformations (one-to-one mappings of a set onto itself)—in this context, the axioms of a group are obvious. In fact, Cayley proved that every group can be considered as a type of permutation group! When is this context provided in a calculus class? Whether complex numbers are received as inaccessible or fascinating depends entirely on how they are presented the first time.

A second avenue in this regard is to provide applications of the material to the outside world: “It is our job as teachers of mathematics to introduce students to this exciting field, and to motivate students to want to study mathematics and to major in it. Applications are a device for achieving this end” [K, 32]. Especially for the engineering majors in the class, seeing how series estimates apply in the real world can be essential to their intrinsic motivation.

Adapt an appropriate level of challenge. It is conventional wisdom that students should be provided with moderately difficult tasks. If the problems are too difficult, they become frustrated, and if they are too easy, they induce boredom or the students begin to believe that the GSI has low expectations. After all, students that “get it” are motivated to learn more. This requires the instructor to know his or her class: their majors, their expectations for the class, their backgrounds, and so on. It also seems to call for continual feedback throughout the course so that the class matches the level of the students in it!

Use group learning effectively. Part of the *raison d’être* for the discussion section is to facilitate interactivity and cooperative learning in a smaller class setting. The belief that students learn better when actively engaged in group work is, for the most part, borne out in the research. “Cooperation among students typically results in higher achievement and greater productivity; more caring, supportive, and committed relationships; and greater psychological health, social competence, and self-esteem” [S, 72]. Research supports the idea that students engaged in group work simply learn and understand more than those who only attend lecture. “Lectures often do not succeed in engaging students in the learning process” [K, 98].

It can be difficult, however, to sell your students on the idea of group work on worksheets, and if small group activities are seen as immaterial to the rest of the class, motivation will suffer precipitously. Consider the following advice: “In general terms, probably the most important thing to do in avoiding difficulties is to see that the students are ‘sold’ on cooperative learning, that they come to believe that this approach will work for them. It is necessary to convince students that more will be accomplished in a more pleasant atmosphere by working in a group than by working individually” [H, 74]. After all, group work is preparation for real life, where projects are accomplished by teams, and cooperative learning contributes to a general level of sociality in the class.

4. CONCLUDING THOUGHTS

The thoughts collected in this paper are only a first attempt at developing some broad strategies to cultivate student motivation in the calculus classroom. Even this task is only one of many faced by a GSI. However, the words of Stephen Krantz bear remembering: “An adequate instructor records the material accurately on the blackboard and then goes home. A truly dynamic instructor interacts with the students, excites their intellectual curiosity, and helps them to discover ideas for themselves” [K, 87]. Indeed, the discussion section is precisely the site where this can occur—a more intimate, interactive setting where students are motivated and actively involved.

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