MATH6991

SCHEMES AND GEOMETRY

Course Outline. Modern algebraic geometry relies on the machinery of schemes, introduced by Grothendieck in his celebrated *EGA*. A scheme can be regarded as a generalisation of projective space, produced by the gluing together not of affine space, but of affine schemes. These affine schemes are "nice" objects, analogous to affine varieties.

The language of schemes is intimately connected with the language of commutative algebra, and it is necessary to be comfortable with the latter before understand the former. We shall see how concepts from classical algebraic geometry can be reformulated in this new language, and how the machinery casts new light on old intuitive concepts. In particular, the notion of "generic point" will be formalised at an early stage of our studies.

Prerequisites. Familiarity with basic algebraic geometry is desirable; confidence with commutative algebra is essential. Any student who is comfortable with the material in the following two books should have no difficulty with this course.

Miles Reid, *Undergraduate algebraic geometry*, London Mathematical Society Student Texts, vol. 12, Cambridge University Press, Cambridge, 1988.

———, *Undergraduate commutative algebra*, London Mathematical Society Student Texts, vol. 29, Cambridge University Press, Cambridge, 1995.

Course Text. This is a reading course, based on:

David Eisenbud and Joe Harris, *The geometry of schemes*, Graduate Texts in Mathematics, vol. 197, Springer-Verlag, New York, 2000.

Additional Reading.

David Eisenbud, *Commutative algebra*, Graduate Texts in Mathematics, vol. 150, Springer-Verlag, New York, 1995, With a view toward algebraic geometry.

Robin Hartshorne, *Algebraic geometry*, Springer-Verlag, New York, 1977, Graduate Texts in Mathematics, No. 52.

David Mumford, *The red book of varieties and schemes*, expanded ed., Lecture Notes in Mathematics, vol. 1358, Springer-Verlag, Berlin, 1999, Includes the Michigan lectures (1974) on curves and their Jacobians, With contributions by Enrico Arbarello.

Kenji Ueno, *Algebraic geometry 1*, Translations of Mathematical Monographs, vol. 185, American Mathematical Society, Providence, RI, 1999.

———, *Algebraic geometry 2*, Translations of Mathematical Monographs, vol. 197, American Mathematical Society, Providence, RI, 2001.