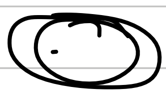
The moduli continuity method for log Fano pairs
(joint with P. Gallardo and C. Spotti)

1. Intro.

Moduli problem: Describe compactification of some families of log Fano pairs with geometric meaning (compact moduli)

GIT → Good: easy to describe (at least with equations for each variety), construct, classify.

GIT → Bad: hard/never modular.

$\overline{M}^{GIT}_{C_4 \subseteq \mathbb{P}^2} \supset$  $\times 2$

2 GIT Hypersurfaces

$D = \{f_d = 0\} \subseteq \mathbb{P}^n \rightsquigarrow [f] \in \mathbb{P}(H^0(\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(d))) = \mathcal{H}$.

Notice $SL_{n+1} \curvearrowright \mathbb{P}^n \rightsquigarrow SL_{n+1} \curvearrowright \mathcal{H}$.

GIT gives a [redacted] compactification of the space of smooth hypersurfaces

$$\overline{M}^{GIT} = \mathbb{P}(H^0(\mathbb{P}^n, \mathcal{O}(d)))^{ss} /\!\!/ SL_{n+1} =$$

$$= \mathrm{Proj}\left(\bigoplus_{m \geq 0} H^0(\mathcal{H}, \mathcal{O}(m))^{SL_{n+1}} \right)$$

= closed "semistable" orbits with finite stabilisers

Well known class for (n, d) small

$(n, d) \in \{(2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5)$ partially Gallardo, $(4,3), (5,3)\}$