Knots, Braids, and the Vogel Algorithm

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Knots
Representing a Knot: The Gauss Code

Gauss Code: -1, +2, -3, +4, +5, +1, -2, +6, +7, +3, -4, -7, -6, -5
Representing a Knot: The Gauss Code

Gauss Code: -1, +2, -3, +4, +5, +I, -2, +6, +7, +3, -4, -7, -6, -5
Representing a Knot: Oriented Gauss Code
Representing a Knot: Oriented Gauss Code
Representing a Knot: Oriented Gauss Code

Oriented Gauss Code:

-1, +2, -3, +4, +5, +1, -2, +6, +7, +3, -4, -7, -6, -5 / - - - - + - +
Seifert Circles
Braids
Braids
Seifertview

[van Wijk, 2005]
Seifertview

[vanWijk, 2005]
Knots ↔ Braids

- Seifertview uses braids
Knots ↔ Braids

- Seifertview uses braids
- Knots and braids are essentially the same things!
Knots ↔ Braids

- Seifertview uses braids
- Knots and braids are essentially the same things!
- Braidgroups, Artin groups, Hecke algebras, etc.
Knots ↔ Braids

- Seifertview uses braids
- Knots and braids are essentially the same things!
- Braidgroups, Artin groups, Hecke algebras, etc.

So we want to convert between knots and braids!
Knots ↔ Braids

\[ \begin{array}{c}
\bigcirc \\
\rightarrow \\
\bigcirc \\
\leftarrow \\
\bigcirc \\
\end{array} \]
Knots ↔ Braids
Braids to Knots
Vogel Algorithm

**Vogel, 1990**

A knot can be brought into suitable form by doing a series of Vogel moves (Reidemeister II moves). The braid word can then easily be read off.
Vogel Algorithm

Vogel, 1990

A knot can be brought into suitable form by doing a series of Vogel moves (Reidemeister II moves). The braid word can then easily be read off.

We have implemented this algorithm in GAP.
Vogel Algorithm: On the Web

The Vogel Algorithm

In Representation of links by braids, a new algorithm (Math. Helvetici 65 (1990), 104-113), Vogel describes an algorithm to manipulate a link diagram in such a way that an equivalent braid word may be read from it (Paper via SpringerLink). Andrew Bartholomew describes the algorithm in great detail in one of his papers as well.

We have implemented the algorithm in GAP, and it can be tested from this very website. In case of problems, please contact Dan Rozemond. One word of warning: For the moment, the algorithm only works for knots, i.e. links with only one component.

Enter a Gauss code and crossing signs below:

Gauss code: $+1\ -2\ +3\ -1\ +2\ -3$  (e.g. $+1\ +2\ -2\ -1$)

Signs: $+++$  (e.g. $+ -$)

Submit Query
Vogel Algorithm: On the Web

The Vogel Algorithm

In *Representation of links by braids, a new algorithm* (Math. Helvetici 65 (1990), 104-113), Vogel describes an algorithm to manipulate a link diagram in such a way that an equivalent braid word may be read from it ([Paper via SpringerLink](https://link.springer.com/article/10.1007/BF01388911)). Andrew Bartholomew describes the algorithm in great detail in one of his papers as well.

We have implemented the algorithm in GAP, and it can be tested from this very website. In case of problems, please contact Dan Roozendan. One word of warning: For the moment, the algorithm only works for knots, i.e. links with only one component.

Executing: `vogel([-1, 2, -3, -1, 1, -3, "+++"));
Result: "B2::aa"

Enter a Gauss code and crossing signs below:

- **Gauss code:** 
  
  
<table>
<thead>
<tr>
<th>+1</th>
<th>-2</th>
<th>+3</th>
<th>-1</th>
<th>+2</th>
<th>-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+1</td>
<td>+2</td>
<td>-2</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **Signs:**
  
  | +++ |
  |+++ |
  
  (e.g. +1  +2  -2  -1 )
  
  (e.g. + - )

[Submit Query]
Knots ↔ Braids

Knot
↕
Oriented Gauss Code
↕
Web page
↕
Braid word
↕
Seifertview
Knotweaver

[Vos, 2007]
Knotweaver

[Vos, 2007]
Knotweaver

[Vos, 2007]
Linking Knotweaver and Seifertview
Linking Knotweaver and Seifertview
Linking Knotweaver and Seifertview
Linking Knotweaver and Seifertview

Knot weaver

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The Vogel Algorithm: A bit more
The Vogel Algorithm: A bit more
The Vogel Algorithm: A bit more
The Vogel Algorithm: A bit more
The Vogel Algorithm: A bit more
Conclusion

- Implementation of the Vogel algorithm
- Link between Knotweaver and Seifertview
Conclusion

- Implementation of the Vogel algorithm
- Link between Knotweaver and Seifertview
- Future options:
  - Smoother connection
  - Multiple components
  - Visualize Vogel algorithm in Knotweaver
Questions?