1. Let \( f(x) = \begin{cases} -x^2 & x < 0 \\ x & 0 \leq x \leq 1 \\ x + 1 & x > 1 \end{cases} \)

(a) Sketch the graph of \( f \). What is the range of \( f \).

You should fill lable the axes and fill put tics on them to indicate the scale. The range of \( f \) is all real numbers except those in the interval \((1, 2]\).

(b) For which values of \( a \) is \( f \) discontinuous at \( x = a \)? Give a reason why \( f \) is not continuous at this value(s).

\( f \) is continuous at all values except \( x = 2 \). At this point, the limit does not exist. Specifically, \( \lim_{x \to a^-} f(x) = 1 \) and \( \lim_{x \to a^+} f(x) = 2 \) are not equal.

(c) See (b).
2. Find the derivatives of the following functions.

(a) \( y = (1 + x^2)^{100} \), \( y' = 100(1 + x^2)^{99}(2x) = 200x(1 + x^2)^{99} \).

(b) \( y = (x + 1)\sin(x) \),

\[
   y' = (1)\sin(x) + (x + 1)(-\cos(x)) \quad \text{product rule}
   \]

\[
   = \sin(x) - (x + 1)\cos(x)
   \]

(c) \( y = \frac{\sqrt{1-x^2}}{x} \), First, let \( f(x) = \sqrt{1-x^2} \) and \( g(x) = x \). Using the chain rule,

\[
   f'(x) = -\frac{x}{\sqrt{1-x^2}},
   \]

and clearly \( g'(x) = 1 \). Using the quotient rule and the above derivatives

\[
   y' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}
   \]

\[
   = -\frac{x^2}{\sqrt{1-x^2}} - \sqrt{1-x^2}
   \]

\[
   = -\frac{1}{x^2\sqrt{1-x^2}}
   \]

(d) \( y = (x^2+1)\sqrt{x^2+2} \), First, let \( f(x) = x^2+1 \) and let \( g(x) = \sqrt{x^2+2} \). Then \( f'(x) = 2x \), and, using the chain rule, \( g'(x) = \frac{2x}{3(2x^2+1)^{\frac{3}{2}}} \). Hence, using the product rule,

\[
   y' = f'(x)g(x) + f(x)g'(x)
   \]

\[
   = 2x\sqrt{x^2+2} + \frac{2x(x^2+1)}{3(2x^2+1)^{\frac{3}{2}}}
   \]

\[
   = \frac{6x^2+2x}{3(2x^2+1)^{\frac{3}{2}}}
   \]

\[
   = \frac{8x^3+14x}{3(2x^2+1)^{\frac{3}{2}}}
   \]

3. Evaluate the following limits:

(a) \( \lim_{x\to 25} \frac{\sqrt{x} - 5}{x - 25} = \lim_{x\to 25} \frac{\sqrt{x} - 5}{(\sqrt{x} - 5)(\sqrt{x} + 5)} = \frac{1}{10} \).

(b) \( \lim_{x\to \infty} \frac{3x^3 - 2x^2}{x^3 + 3x - 1} = 3 \).

(c) \( \lim_{x\to 2} \frac{x^2 - 4}{x^2 - 4x + 4} = \lim_{x\to 2} \frac{(x - 2)(x + 2)}{(x - 2)^2} = \lim_{x\to 2} \frac{(x + 2)}{(x - 2)} = +\infty \)
4. Let \( f(x) = \sqrt{2x+1} \) and \( g(x) = x^2 \).

   (a) The domain of \( f \) is all real numbers \( x \) such that \( x \geq -1/2 \).
   
   (b) \( f(g(x)) = \sqrt{2x^2 + 1} \)

   (c) The domain of \( f(g(x)) \) is all real numbers.

5. Let \( f(x) = x^3 - 3x^2 - 8x \).

   (a) Find the equation of the tangent line to the curve \( y = f(x) \) at the point \((1, -10)\).

   This is the line with slope \( m = f'(1) = 3(1)^2 - 6(1) - 8 = -11 \) passing through the point \((1, -10)\). An equation for this line is
   \[ y + 10 = -11(x - 1) \]

   (b) The tangent line has slope one whenever \( f'(x) = 1 \). Since \( f'(x) = 3x^2 - 6x - 8 \), this occurs when \( x = 3 \) or \( x = -1 \).

6. Compute \( f'(x) \) from the definition of the derivative when \( f(x) = 3x - x^2 \).

   \[
   f'(x) = \lim_{h\to0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to0} \frac{(3(x+h) - (x+h)^2) - (3x - x^2)}{h} = \lim_{h\to0} \frac{3x + 3h - x^2 - 2xh - h^2 - 3x + x^2}{h} = \lim_{h\to0} \frac{(3 - 2x)h - h^2}{h} = \lim_{h\to0} ((3 - 2x) - h) = 3 - 2x
   \]