

STUDENT'S NAME: \_\_\_\_\_ ID #: \_\_\_\_\_

PROFESSOR'S NAME: \_\_\_\_\_ SECTION #: \_\_\_\_\_

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1003

MID-TERM TEST

WEDNESDAY, OCTOBER 29, 2003

MARKS

1. Let  $f(x) = \begin{cases} -x^2 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ x + 1 & \text{if } x > 1 \end{cases}$ .

- (3) (a) Sketch the graph of  $f$ . What is the range of  $f$ ?
- (2) (b) For which values  $a$  is  $f$  discontinuous at  $x = a$ ? Give a reason why  $f$  is not continuous at this value (or values).
- (2) (c) At each value  $a$  where  $f$  is discontinuous, determine both  $\lim_{x \rightarrow a^+} f(x)$  and  $\lim_{x \rightarrow a^-} f(x)$ .

2. Find the derivatives of the following functions. You do not need to simplify your answers.

(4) (a)  $y = (1 + x^2)^{100}$

(4) (b)  $y = (x + 1) \sin x$

(4) (c)  $y = \frac{\sqrt{1 - x^2}}{x}$

(4) (d)  $y = (x^2 + 1) \sqrt[3]{x^2 + 2}$

3. Evaluate the following limits:

(3) (a)  $\lim_{x \rightarrow 25} \frac{\sqrt{x} - 5}{x - 25}$

(3) (b)  $\lim_{x \rightarrow \infty} \frac{3x^3 - 2x^2}{x^3 + 3x - 1}$

(3) (c)  $\lim_{x \rightarrow 2^+} \frac{x^2 - 4}{x^2 - 4x + 4}$

4. Let  $f(x) = \sqrt{2x + 1}$  and  $g(x) = x^2$ . Find

(2) (a) the domain of  $f$ ;

(2) (b)  $f \circ g(x)$ ;

(2) (c) the domain of  $f \circ g$ .

5. Let  $f(x) = x^3 - 3x^2 - 8x$ .

(3) (a) Find the equation of the tangent to the curve  $y = f(x)$  at the point  $(1, -10)$ .

(3) (b) Find the values of  $x$  where the tangent line to  $y = f(x)$  has slope 1.

(2) 6. (a) State the definition of the derivative of the function  $f(x)$ .

(4) (b) Let  $f(x) = 3x - x^2$ . Use the definition of the derivative to find  $f'(x)$ . (We all know  $f'(x) = 3 - 2x$ . You must use the definition to receive credit!)