**MATH1003**

# QUIZ 3 – MOCK MID-TERM

*This quiz has five questions, with each question worth 5 marks.*
*The quiz lasts for one-and-a-half hours. No calculators, books etc. are allowed.*

**1.** The function $f : (-7, 7) \to \mathbb{R}$ is drawn in Figure 1 for $0 \leq x < 7$. Taking care to

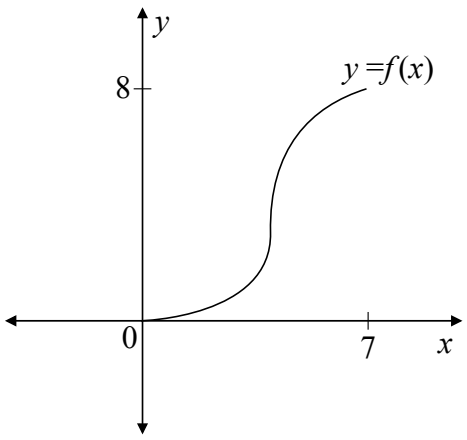


FIGURE 1. The graph of $y = f(x)$ for $0 \leq x < 7$.

label your graphs clearly, sketch $y = f(x)$ for all values of $x$ in the domain of $f$, assuming that:

(i) $f$ is an even function;
(ii) $f$ is an odd function;
(iii) $f$ is neither even nor odd.

**2.** The rational function $f(x)$ and the polynomial $g(x)$ are defined by:

$$f(x) = \frac{1}{x+1}, \qquad g(x) = x^3 + 1.$$

(i) What are the domains of $f$ and $g$?
(ii) Find an expression for $f \circ g$, and state the domain.
(iii) Find an expression for $g \circ f$, and state the domain.
(iv) What can we say about the continuity of $f \circ g$ and $g \circ f$?

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**3.** The piecewise function $\varphi : \mathbb{R} \to \mathbb{R}$ is given by:

$$\varphi(x) = \begin{cases} x+1, & \text{if } x \leq 1; \\ x^2, & \text{otherwise.} \end{cases}$$

(i) Sketch the graph of $y = \varphi(x)$.

(ii) By calculating the left and right limits separately as $x \to 1$, or otherwise, calculate $\lim_{x\to 1} \varphi(x)$ or explain why no such limit exists.

(iii) State the definition of what it means for a function to be continuous at a point $a$. When is $\varphi$ continuous?

**4.** Let $F : \mathbb{R} \to \mathbb{R}$ be the curve given by $F(x) = \sqrt{x^2+5}$.

(i) State the definition of what it means for a function to be differentiable.

(ii) From the definition in (i), calculate $F'(x)$.

(iii) Verify that the point $(2, 3)$ lies on the curve $y = F(x)$.

(iv) Find the equation of the line tangent to $y = F(x)$ at $(2, 3)$.

**5.** Evaluate the following limits, or give a reason why they do not exist:

(i) $\displaystyle\lim_{x\to 1} \frac{x^2-1}{x-1}$,

(ii) $\displaystyle\lim_{x\to\infty} \frac{1+x}{\sqrt{2x^2+1}}$,

(iii) $\displaystyle\lim_{x\to\infty} e^{-x}\cos x$.