MATH1003
QUIZ 2, QUESTION 2 (II)

Question 2(ii) was a mistake on my part, and is exceptionally difficult to do without resorting to a calculator (in which case it is trivial). Here is one possible solution.

Question. Show that there is a solution to $\ln x = e^{-x}$ in the interval $(1,2)$.

Solution. Note that $e < 4$ and so $\sqrt{e} < 2$. Hence it is sufficient to show that there is a solution in the smaller interval $(1, \sqrt{e})$.

Observe that $\ln 1 = 0 < e^{-1}$. If we can show that $\ln \sqrt{e} > e^{-\sqrt{e}}$ then we are done, via the Intermediate Value Theorem.

Since e^x is strictly increasing, we have that $e^{-\sqrt{e}} < e^{-1}$. Hence it is enough to prove that:

$$\ln \sqrt{e} > e^{-1}.$$

Suppose for a contradiction that $\ln \sqrt{e} \leq e^{-1}$. By the Laws of Logarithms we have that:

$$\begin{aligned} & \ln \sqrt{e} \leq e^{-1} \\ \Rightarrow \quad & \frac{1}{2} \ln e \leq e^{-1} \\ \Rightarrow \quad & \frac{1}{2} \leq e^{-1} \\ \Rightarrow \quad & e \leq 2 \end{aligned}$$

But this is absurd (since $e > 2$). Hence our assumption that $\ln \sqrt{e} \leq e^{-1}$ must be false, and thus $\ln \sqrt{e} > e^{-1}$, as required.

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