MATH1003

ASSIGNMENT 9

Suggested practice questions (the answers are in the back of the textbook):

- *§4.2; 1, 3, 7.*
- *§4.5; 1, 9, 17, 19, 25, 41, 71.*
- *§4.6; 11, 13.*

1. Let $f : [0, 2] \to \mathbb{R}$ be given by $f(x) = x^3 + x - 1$. Verify that the function satisfies the hypotheses of the Mean Value Theorem. Find all numbers c that satisfy the conclusion of the Mean Value Theorem.

2. Prove the following result:

Proposition. *Let f and g be continuous on $[a, b]$ and differentiable on (a, b). Suppose also that $f(a) = g(a)$ and $f'(x) < g'(x)$ for $a < x < b$. Then $f(b) < g(b)$.*

(Hint: Apply the Mean Value Theorem to the function $h = f - g$.)

3. The graph of the first derivative f' of a function f is given in Figure 1.

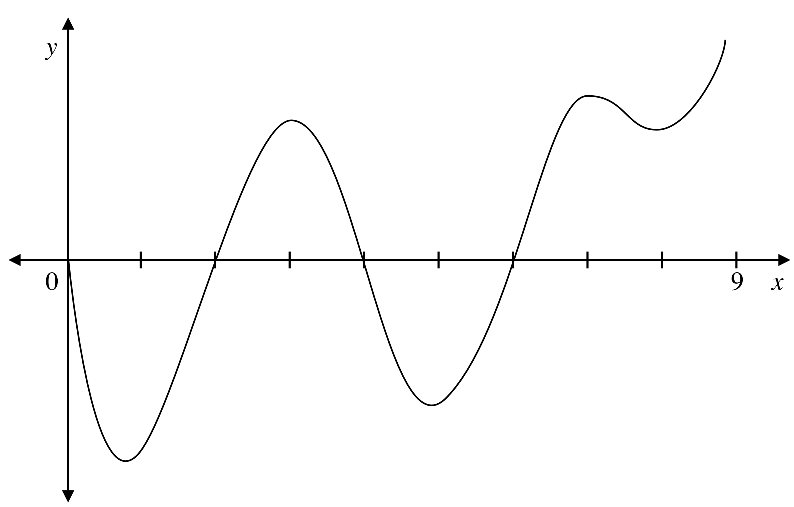

FIGURE 1. The graph of $y = f'(x)$.

(i) On what intervals is f increasing?

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(ii) At which values of x does f have a local minimum or maximum?
(iii) On what intervals if f concave upwards or concave downwards?
(iv) What are the x-coordinates of the inflection points of f?

Remember to justify your answers.

4. Let $B(x) = 3x^{2/3} - x$.

(i) Find the intervals of increase or decrease.
(ii) Find the local minimum and maximum values.
(iii) Find the intervals of concavity and the inflection points.
(iv) Using these results, sketch a graph of $y = B(x)$.

5. Consider the function $y = \dfrac{x^2 - 2}{x^4}$.

(i) Find the points of intersection with the axes.
(ii) Find any asymptotes.
(iii) What happens as $x \to \pm\infty$?
(iv) Find the local minima and maxima, the intervals of concavity, and the points of inflection.
(v) Sketch the function.