1. Let \( f : [0, 2] \to \mathbb{R} \) be given by \( f(x) = x^3 + x - 1 \). Verify that the function satisfies the hypotheses of the Mean Value Theorem. Find all numbers \( c \) that satisfy the conclusion of the Mean Value Theorem.

2. Prove the following result:

   **Proposition.** Let \( f \) and \( g \) be continuous on \([a, b]\) and differentiable on \((a, b)\). Suppose also that \( f(a) = g(a) \) and \( f'(x) < g'(x) \) for \( a < x < b \). Then \( f(b) < g(b) \).

   (Hint: Apply the Mean Value Theorem to the function \( h = f - g \).)

3. The graph of the first derivative \( f' \) of a function \( f \) is given in Figure 1.

![Figure 1](http://erdos.math.unb.ca/~kasprzyk/kasprzyk@unb.ca)

   **Figure 1.** The graph of \( y = f'(x) \).

   (i) On what intervals is \( f \) increasing?
(ii) At which values of $x$ does $f$ have a local minimum or maximum?

(iii) On what intervals if $f$ concave upwards or concave downwards?

(iv) What are the $x$-coordinates of the inflection points of $f$?

Remember to justify your answers.

4. Let $B(x) = 3x^{2/3} - x$.

   (i) Find the intervals of increase or decrease.

   (ii) Find the local minimum and maximum values.

   (iii) Find the intervals of concavity and the inflection points.

   (iv) Using these results, sketch a graph of $y = B(x)$.

5. Consider the function $y = x^2 - 2x^4$.

   (i) Find the points of intersection with the axes.

   (ii) Find any asymptotes.

   (iii) What happens as $x \to \pm\infty$?

   (iv) Find the local minima and maxima, the intervals of concavity, and the points of inflection.

   (v) Sketch the function.