**MATH1003**
# ASSIGNMENT 4

*Suggested practice questions (the answers are in the back of the textbook):*

- *§3.1; 33, 35, 53, 61.*
- *§3.2; 13, 27, 35, 45.*

**1.** If $f$ is a differentiable function, find an expression for the derivative of each of the following functions:

(i) $y = xf(x)$,

(ii) $y = \dfrac{f(x)}{x}$,

(iii) $y = \dfrac{x^2}{f(x)}$,

(iv) $y = \dfrac{1 + xf(x)}{\sqrt{x}}$.

**2.** (i) Use the Product Rule twice to prove that if $f$, $g$, and $h$ are differentiable then:

$$(fgh)' = f'gh + fg'h + fgh'.$$

(ii) Hence or otherwise, show that:

$$\frac{d}{dx} f(x)^3 = 3f(x)^2 f'(x).$$

(iii) Calculate the derivative of $y = \tan^3 x$.

**3.** (i) Let $g$ be a differentiable function. By using the Quotient Rule, prove that:

$$\frac{d}{dx}\frac{1}{g(x)} = -\frac{g'(x)}{(g(x))^2}.$$

(ii) Using the result given in (i), calculate the derivative of:

$$y = \frac{1}{x^4 + x^2 + 1}.$$

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(iii) Prove that the Power Rule is valid for all negative integers. That is, prove that

$$\frac{d}{dx}x^{-n} = -nx^{-n-1}$$

for all positive integers $n$. (Hint: Use (i).)

**4.** A tangent line is drawn to the hyperbola $xy = c$ at a point $P$.

(i) Show that the midpoint of the line segment cut from this tangent line by the coordinate axes is $P$.

(ii) Show that the triangle formed by the tangent line and the coordinate axes always has the same area, no matter where $P$ is located on the hyperbola.

(Hint: Sketch the hyperbola, mark a point $P$ and draw the tangent line.)