1. Be careful! Since $1 - \sin \theta \to 0$ and $\csc \theta \to 1$ as $\theta \to \pi/2$, L’Hôpital’s Rule does not apply. But this isn’t a problem, since we need only use the Laws of Limits:

$$\lim_{\theta \to \pi/2} \frac{1 - \sin \theta}{\csc \theta} = 0$$

2. Proposition. For any $\rho > 0$,

$$\lim_{x \to \infty} \frac{\ln x}{x^\rho} = 0$$

Proof. Observe that both $\ln x \to \infty$ and $x^\rho \to \infty$ as $x \to \infty$. Hence we can apply L’Hôpital’s Rule to obtain that:

$$\lim_{x \to \infty} \frac{\ln x}{x^\rho} = \lim_{x \to \infty} \frac{1/x}{\rho x^{\rho-1}}$$

$$= \lim_{x \to \infty} \frac{1}{\rho x^\rho}$$

$$= 0.$$ 

3. (i) Let $f(x) = x^3 + x^2 - x$. Then:

$$\frac{dy}{dx} = 3x^2 + 2x - 1$$

$$= (3x - 1)(x + 1),$$

which is defined everywhere, and equals zero when $x = -1$ or $x = 1/3$. Hence the critical numbers are $x = -1$ and $x = 1/3$.

(ii) Let $g(\theta) = 4\theta - \tan \theta$. This is undefined when $\theta = (2k + 1)\pi/2$, where $k \in \mathbb{Z}$. The derivative is given by:

$$g'(\theta) = 4 - \sec^2 \theta.$$
This is defined whenever $g$ is defined, and is zero when:

\[
\sec^2 \theta = 4
\]
\[
\Rightarrow \cos^2 \theta = \frac{1}{4}
\]
\[
\Rightarrow \cos \theta = \pm \frac{1}{2}
\]
\[
\Rightarrow \theta = 2k\pi \pm \frac{\pi}{3}, \quad \text{where} \quad k \in \mathbb{Z}
\]
\[
\Rightarrow \theta = k\pi \pm \frac{\pi}{3}.
\]

Hence the critical numbers are at $k\pi \pm \pi/3$, for all $k \in \mathbb{Z}$.

**4.**  
(i) First note that $f$ is continuous on this interval. We have that:

\[
f'(x) = 3x^2 - 12x + 9
\]
\[
= 3(x - 1)(x - 3).
\]

This is defined for all $x$ in $(-1, 4)$, and is zero when $x = 3$ or $x = 1$. Hence the critical values are $x = 3$ and $x = 1$. The Closed Interval Method tells us that the global minimum and global maximum values of $f$ on $[-1, 4]$ are given by one of the critical values or by the end points.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$-14$</td>
</tr>
<tr>
<td>$1$</td>
<td>$6$</td>
</tr>
<tr>
<td>$3$</td>
<td>$2$</td>
</tr>
<tr>
<td>$4$</td>
<td>$6$</td>
</tr>
</tbody>
</table>

Hence the global maximum value is given by 6, and occurs when $x = 1$ and when $x = 4$. The global minimum value is $-14$, and occurs when $x = -1$.

(ii) Differentiating $f$ gives:

\[
f'(x) = 6x^2 - 6x - 12
\]
\[
= 6(x^2 - x - 2)
\]
\[
= 6(x + 1)(x - 2).
\]

This is zero when $x = -1$ and when $x = 2$; these are the critical values. The global maximum and global minimum values of $f$ occur at the critical values,
or at the end points of the domain. Hence we need to calculate:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2$</td>
<td>$-3$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$8$</td>
</tr>
<tr>
<td>$2$</td>
<td>$-19$</td>
</tr>
<tr>
<td>$3$</td>
<td>$-8$</td>
</tr>
</tbody>
</table>

The global maximum value is $8$ (when $x = -1$), and the global minimum value is $-19$ (when $x = 2$).

(iii) Differentiating $f$ gives:

$$f'(x) = 3 \times 2x \times (x^2 - 1)^2$$

$$= 6x(x^2 - 1)^2.$$  

This is zero when $x = 0$ and when $x = \pm1$; these are the critical values. The global maximum and global minimum values of $f$ occur at the critical values, or at the end points of the domain. Hence we need to calculate:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$0$</td>
<td>$1$</td>
</tr>
<tr>
<td>$1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$2$</td>
<td>$9$</td>
</tr>
</tbody>
</table>

The global maximum value is $9$ (when $x = 2$), and the global minimum value is $0$ (when $x = -1$ and when $x = 1$).

(iv) Differentiating $f$ gives:

$$f'(x) = e^{-x} - xe^{-x}$$

$$= (1 - x)e^{-x}.$$  

This is zero when $x = 1$; this is the only critical value. The global maximum and global minimum values of $f$ occur at a critical value, or at the end points of the domain. Hence we need to calculate:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$1$</td>
<td>$e^{-1}$</td>
</tr>
<tr>
<td>$2$</td>
<td>$2e^{-2}$</td>
</tr>
<tr>
<td></td>
<td>$3$</td>
</tr>
</tbody>
</table>
The global minimum value is 0 (when $x = 0$). Observe that:

$$e^{-1} < 2e^{-2}$$

$$\Rightarrow \quad e < 2$$

which is a contradiction. Hence $e^{-1} > 2e^{-2}$ and so the global maximum value is $e^{-1}$ (when $x = 1$).