MATH1003

ASSIGNMENT 5
ANSWERS

1. (i)

$$\begin{aligned}\frac{d}{dx}\sec x = \frac{d}{dx}\frac{1}{\cos x} &= -\frac{1}{\cos^2 x}\frac{d}{dx}\cos x \\ &= \frac{1}{\cos^2 x}\sin x \\ &= \sec x \tan x.\end{aligned}$$

(ii)

$$\begin{aligned} y &= \frac{x^2}{\cos x} \\ &= x^2 \sec x \\ \Rightarrow \quad \frac{dy}{dx} &= 2x \times \sec x + x^2 \times \sec x \tan x \\ &= x \sec x (2 + x \tan x).\end{aligned}$$

(iii)

$$\begin{aligned}\frac{dy}{dx} &= \sec x \tan x \times (x - \cot x) + \sec x \times \frac{d}{dx}(x - \cot x) \\ &= x \sec x \tan x - \sec x + \sec x \Big(1 - \frac{d}{dx}\frac{\cos x}{\sin x}\Big) \\ &= x \sec x \tan x - \sec x + \sec x \Big(1 + \frac{\sin^2 x + \cos^2 x}{\sin^2 x}\Big) \\ &= x \sec x \tan x - \sec x + \sec x \Big(1 + \frac{1}{\sin^2 x}\Big) \\ &= x \sec x \tan x + \sec x \csc^2 x \\ &= \sec x (x \tan x + \csc^2 x).\end{aligned}$$

(iv) First, note that:

$$\frac{d}{dx}\sin(\sin x) = \cos(\sin x)\cos x.$$

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Setting $u(x) = \sin(\sin x)$:

$$\frac{d}{dx} \sin u(x) = u'(x) \cos u(x).$$

Hence the answer is:

$$\frac{dy}{dx} = \cos x \cdot \cos(\sin x) \cdot \cos(\sin(\sin x)).$$

(v) Begin by recalling that:

$$\frac{d}{dx} \csc x = -\csc x \cot x.$$

Hence:

$$\begin{aligned} \frac{d}{dx}(\csc x)^4 &= 4(\csc x)^3 \times -\csc x \cot x \\ &= -4(\csc x)^4 \cot x. \end{aligned}$$

Using the Quotient Rule we obtain:

$$\begin{aligned} \frac{dy}{dx} &= \frac{-4(\csc x)^4 \cot x \times 2x^2 - (\csc x)^4 \times 4x}{4x^4} \\ &= -(\csc x)^4 \frac{2x \cot x + 1}{x^3}. \end{aligned}$$

2. Let $y = \sin 2x - 2\sin x$. Then:

$$\frac{dy}{dx} = 2\cos 2x - 2\cos x.$$

This is zero when:

$$\begin{aligned} & 2(\cos 2x - \cos x) = 0, \\ \Rightarrow \quad & 2\cos^2 x - \cos x - 1 = 0, \\ \Rightarrow \quad & (2\cos x + 1)(\cos x - 1) = 0, \\ \Rightarrow \quad & \cos x = -\frac{1}{2} \text{ or } \cos x = 1. \end{aligned}$$

Hence $x = 2k\pi$, or $x = 2(k+1)\pi \pm \pi/3$, where $k \in \mathbb{Z}$.

3. (i) Let $u = e^x$, so that $F(x) = f(u)$. By the Chain Rule,

$$\begin{aligned} F'(x) &= \frac{df}{du}\frac{du}{dx} \\ &= f'(u)e^x \\ &= f'(e^x)e^x. \end{aligned}$$

(ii) Let $u = f(x)$, so that $F(x) = e^u$. By the Chain Rule,

$$\begin{aligned} F'(x) &= \frac{d}{du}(e^u)\frac{du}{dx} \\ &= e^u f'(x) \\ &= e^{f(x)} f'(x) \\ &= F(x) f'(x). \end{aligned}$$

(iii) Let $u = x^\alpha$, so that $F(x) = f(u)$. By the Chain Rule,

$$\begin{aligned} F'(x) &= \frac{df}{du}\frac{du}{dx} \\ &= f'(u) \times \alpha x^{\alpha-1} \\ &= \alpha x^{\alpha-1} f'(x^\alpha). \end{aligned}$$

(iv) Let $u = f(x)$, so that $F(x) = u^\alpha$. By the Chain Rule,

$$\begin{aligned} F'(x) &= \frac{d}{du}u^\alpha \frac{du}{dx} \\ &= \alpha u^{\alpha-1} \times f'(x) \\ &= \alpha f(x)^{\alpha-1} f'(x). \end{aligned}$$

4. (i) Let $y = e^{-rx}$. Then $y' = -re^{-rx}$ and $y'' = r^2 e^{-rx}$.

(ii) Substituting in we get:

$$\begin{aligned} &r^2 e^{-rx} + 2r \times -re^{-rx} + r^2 \times e^{-rx} \\ &= r^2 e^{-rx} - 2r^2 e^{-rx} + r^2 e^{-rx} \\ &= 0. \end{aligned}$$

(iii) If we set $r = -3$ we see from (ii) that $y = e^{-(-3)x} = e^{3x}$ satisfies:

$$y'' - 6y' + 9y = 0.$$

If we set $y = e^{3x} + 2$ then y' and y'' remain unchanged (the constant vanishes when we differentiate), and we obtain:

$$y'' - 6y + 9y = 0 + 9 \times 2 = 18,$$

as desired.