MATH1003

ASSIGNMENT 4
ANSWERS

1. (i)

$$\begin{aligned}\frac{dy}{dx} &= 1 \times f(x) + x \times f'(x)\\ &= f(x) + xf'(x).\end{aligned}$$

(ii)

$$\begin{aligned}\frac{dy}{dx} &= \frac{f'(x) \times x - f(x) \times 1}{x^2}\\ &= \frac{f'(x)}{x} - \frac{f(x)}{x^2}.\end{aligned}$$

(iii)

$$\frac{dy}{dx} = 2xf(x) + x^2 f'(x).$$

(iv)

$$\begin{aligned}\frac{dy}{dx} &= \frac{(f(x) + xf'(x))\sqrt{x} - (1/2)(1 + xf(x))x^{-1/2}}{x},\\ &= \frac{(f(x) + xf'(x))x - (1/2)(1 + xf(x))}{x^{3/2}},\\ &= \frac{2x^2 f'(x) + xf(x) - 1}{2x^{3/2}}.\end{aligned}$$

2. (i)

$$\begin{aligned}(fgh)' &= (f(gh))'\\ &= f' \times (gh) + f \times (gh)'\\ &= f' \times (gh) + f \times (g' \times h + g \times h')\\ &= f'gh + fg'h + fgh'.\end{aligned}$$

(ii) This is immediate from (i), setting $f = g = h$.

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(iii) We know that:

$$\frac{d}{dx}\tan(x) = \sec^2(x).$$

Hence from our answer to (ii) we obtain:

$$\frac{dy}{dx} = 3\tan^2(x)\sec^2(x).$$

3. (i) By the Quotient Rule we have:

$$\begin{aligned}\frac{d}{dx}\frac{1}{g(x)} &= \frac{0 \times g(x) - 1 \times g'(x)}{g(x)^2}, \\ &= -\frac{g'(x)}{g(x)^2}.\end{aligned}$$

(ii) Setting $g(x) = x^4 + x^2 + 1$ we obtain:

$$\frac{d}{dx}\frac{1}{x^4 + x^2 + 1} = -\frac{4x^3 + 2x}{(x^4 + x^2 + 1)^2}.$$

(iii) If we set $g(x) = 1/x^n$ we see that:

$$\begin{aligned}\frac{d}{dx}\frac{1}{x^n} &= -\frac{nx^{n-1}}{x^{2n}}, \\ &= -nx^{n-1-2n}, \\ &= -nx^{-n-1}.\end{aligned}$$

This verifies the Power Rule for negative integers.

4. (i) Consider the hyperbola $xy = c$. This has derivative:

$$\begin{aligned} y + x\frac{dy}{dx} &= 0, \\ \Rightarrow \quad \frac{dy}{dx} &= -\frac{y}{x}.\end{aligned}$$

Let P be the point $(a, c/a)$ on this hyperbola. The tangent line at P has equation:

$$\begin{aligned} y - \frac{c}{a} &= -\frac{c}{a^2}(x - a), \\ \Rightarrow \quad y &= \frac{c}{a^2}(2a - x).\end{aligned}$$

This tangent intersects the x-axis when $y = 0$. Hence $x = 2a$. It intersects the y-axis when $x = 0$, giving $y = 2c/a$. The line segment connecting $(2a, 0)$ and $(0, 2c/a)$ has midpoint:

$$\frac{1}{2}\left(2a, \frac{2c}{a}\right) = \left(a, \frac{c}{a}\right) = P.$$

(ii) The triangle formed by the coordinate axes and the tangent has vertices at $(0, 0)$, $(2a, 0)$, and $(0, 2c/a)$. This has width $2a$ and height $2c/a$, and so has area:

$$\frac{1}{2} \times 2a \times \frac{2c}{a} = 2c.$$

Hence the area is independent of the value of a, and so does not depend on our choice of P.