Equations for Modular Curves

Enrique González Jiménez

Departamento de Matemáticas
Universidad Autónoma de Madrid

Magma 2006
Berlin, 31st July 2006

Equations for Modular Curves

1. Modularity
   - Departure point
   - Generalization
   - Newness
2. Finiteness Results
   - $g = 0, 1$
   - $g = 2$
   - $g \geq 2$
3. Computationality
   - Hyperelliptic Curves
   - Non-Hyperelliptic Curves
   - Final Remark

Equations for Modular Curves

Departure point

Shimura-Taniyama-Weil Conjecture

Let $C_{/\mathbb{Q}}$ be an elliptic curve. Then

$$\exists \pi_{/\mathbb{Q}} : X_0(N) \longrightarrow\!\!\!\!\!\rightarrow C\,,$$

for some positive integer N.

Theorem

Let $C_{/\mathbb{Q}}$ be an elliptic curve. Then

$$\exists\, \pi_{/\mathbb{Q}} : X_0(N) \twoheadrightarrow C\,,$$

for some positive integer N.

A. Wiles,
Modular elliptic curves and Fermat's Last Theorem,
Ann. Math. **141** (1995), 443–551.

C. Breuil, B. Conrad, F. Diamond, and R. Taylor,
On the modularity of elliptic curves over $\mathbb{Q}$: wild 3-adic exercises,
J. Amer. Math. Soc. **14(4)** (2001), 843–939.

Theorem

Let $C_{/\mathbb{Q}}$ be an elliptic curve. Then

$$\exists \pi_{/\mathbb{Q}} : X_0(N) \twoheadrightarrow C,$$

for some positive integer N.

A. Wiles,
Modular elliptic curves and Fermat's Last Theorem,
Ann. Math. **141** (1995), 443–551.

C. Breuil, B. Conrad, F. Diamond, and R. Taylor,
On the modularity of elliptic curves over $\mathbb{Q}$: wild 3-adic exercises,
J. Amer. Math. Soc. **14(4)** (2001), 843–939.

Theorem

Let $C_{/\mathbb{Q}}$ be an elliptic curve. Then

$$\exists \pi_{/\mathbb{Q}} : X_0(N) \longrightarrow\!\!\!\!\rightarrow C,$$

for some positive integer N.

A. Wiles,
Modular elliptic curves and Fermat's Last Theorem,
Ann. Math. **141** (1995), 443–551.

C. Breuil, B. Conrad, F. Diamond, and R. Taylor,
On the modularity of elliptic curves over $\mathbb{Q}$: wild 3-adic exercises,
J. Amer. Math. Soc. **14(4)** (2001), 843–939.

Generalization

$$X_0(N) \xrightarrow{\ \ \pi\ \ } C$$

Generalization

$$X_1(N) \twoheadrightarrow X_0(N) \xrightarrow{\pi}\!\!\twoheadrightarrow C$$

Generalization

$$X_1(N) \xrightarrow{\pi} C$$

Generalization

$$X_1(N) \xrightarrow{\pi} C$$

- Modularity of elliptic curves over $\overline{\mathbb{Q}}$. MAGMA package in progress
- Modularity of curves over $\mathbb{Q}$ of genus $g > 1$.
- Modularity of curves over $\overline{\mathbb{Q}}$ of genus $g > 1$.

Generalization

$$X_1(N) \xrightarrow{\pi} C$$

- Modularity of elliptic curves over $\overline{\mathbb{Q}}$. MAGMA package in progress
- Modularity of curves over $\mathbb{Q}$ of genus $g > 1$.
- Modularity of curves over $\overline{\mathbb{Q}}$ of genus $g > 1$.

Generalization

$$X_1(N) \xrightarrow{\pi} C$$

- Modularity of elliptic curves over $\overline{\mathbb{Q}}$. MAGMA package in progress
- Modularity of curves over $\mathbb{Q}$ of genus $g > 1$.
- Modularity of curves over $\overline{\mathbb{Q}}$ of genus $g > 1$.

Generalization

$$X_1(N) \xrightarrow{\pi} C$$

- Modularity of elliptic curves over $\overline{\mathbb{Q}}$. MAGMA package in progress
- Modularity of curves over $\mathbb{Q}$ of genus $g > 1$.
- Modularity of curves over $\overline{\mathbb{Q}}$ of genus $g > 1$.

Generalization

$$X_1(N) \xrightarrow{\pi} C$$

- Modularity of elliptic curves over $\overline{\mathbb{Q}}$. MAGMA package in progress
- Modularity of curves over $\mathbb{Q}$ of genus $g > 1$.
- Modularity of curves over $\overline{\mathbb{Q}}$ of genus $g > 1$.

Generalization

$$X_1(N) \xrightarrow{\pi} C$$

-
- Modularity of curves over $\mathbb{Q}$ of genus $g > 1$.
-

Definition

$C_{/\mathbb{Q}}$ is a *modular curve of level N* if

$$\exists \pi_{/\mathbb{Q}} : X_1(N) \twoheadrightarrow C \, .$$

Definition

$C_{/\mathbb{Q}}$ is a *modular curve of level N* if

$$\exists \pi_{/\mathbb{Q}} : X_1(N) \twoheadrightarrow C\,.$$

$$X_1(N) \xrightarrow{\ \pi\ } \!\!\!\!\twoheadrightarrow C$$

Definition

$C_{/\mathbb{Q}}$ is a *modular curve of level N* if

$$\exists \pi_{/\mathbb{Q}} : X_1(N) \twoheadrightarrow C\,.$$

$$J_1(N) \xrightarrow{\pi_*} J(C)$$

$$X_1(N) \xrightarrow{\pi} C$$

Definition

$C_{/\mathbb{Q}}$ is a *modular curve of level N* if

$$\exists \pi_{/\mathbb{Q}} : X_1(N) \twoheadrightarrow C\,.$$

In that case, $J(C)$ is modular of level N, since we have

$$J_1(N) \xrightarrow{\pi_*} J(C)$$

$$X_1(N) \xrightarrow{\pi} C$$

Definition

$C_{/\mathbb{Q}}$ is a *modular curve of level N* if

$$\exists \pi_{/\mathbb{Q}} : X_1(N) \twoheadrightarrow C\,.$$

In that case, $J(C)$ is modular of level N, since we have

$$\begin{tikzcd}
J_1(N) \arrow[r, two heads, "\pi_*"] & J(C) \\
X_1(N) \arrow[u, hook] \arrow[r, two heads, "\pi"] & C \arrow[u, hook]
\end{tikzcd}$$

Definition

$C_{/\mathbb{Q}}$ is a *modular curve of level N* if

$$\exists \pi_{/\mathbb{Q}} : X_1(N) \twoheadrightarrow C\,.$$

In that case, $J(C)$ is modular of level N, since we have

$$\begin{tikzcd} J_1(N) \arrow[r, two heads, "\pi_*"] & J(C) \\ X_1(N) \arrow[u, hook] & C \arrow[u, hook] \end{tikzcd}$$

The converse is not true in general.

Definition

$C_{/\mathbb{Q}}$ is a *modular curve of level N* if

$$\exists \pi_{/\mathbb{Q}} : X_1(N) \twoheadrightarrow C\,.$$

In that case, $J(C)$ is modular of level N, since we have

$$\begin{tikzcd} J_1(N) \arrow[r, two heads, "\pi_*"] & J(C) \\ X_1(N) \arrow[u, hook] & C \arrow[u, hook] \end{tikzcd}$$

The converse is not true in general.

Definition

$C_{/\mathbb{Q}}$ is a modular curve of level N if

$$X_1(N) \xrightarrow{\pi_{/\mathbb{Q}}} C$$

Definition

$C_{/\mathbb{Q}}$ is a modular curve of level N if

$$\begin{tikzcd} J_1(N) \arrow[r, two heads, "\pi_*"] & J(C) \\ X_1(N) \arrow[u, hook] \arrow[r, two heads, "\pi"] & C \arrow[u, hook] \end{tikzcd}$$

Definition

$C_{/\mathbb{Q}}$ is a *new modular curve of level* N if

$$\begin{tikzcd}
J_1(N) \arrow[rr, two heads, "\pi_*"] \arrow[dr, two heads] & & J(C) \\
& J_1(N)^{\text{new}} \arrow[ur, two heads] & \\
X_1(N) \arrow[uu, hook] \arrow[rr, two heads, "\pi"] & & C \arrow[uu, hook]
\end{tikzcd}$$

Definition

$C_{/\mathbb{Q}}$ is a *new modular curve of level N* if

$$\begin{tikzcd}
J_1(N) \arrow[dr, two heads] & & J(C) \\
& J_1(N)^{\text{new}} \arrow[ur, two heads] & \\
X_1(N) \arrow[uu, hook] \arrow[rr, two heads, "\pi"] & & C \arrow[uu, hook]
\end{tikzcd}$$

Then

$$\pi^* H^0(C, \Omega^1) \hookrightarrow S_2(N)^{\text{new}} \frac{dq}{q}$$

Definition

$C_{/\mathbb{Q}}$ is a *new modular curve of level N* if

$$\begin{tikzcd}
J_1(N) \arrow[dr, twoheadrightarrow] & & J(C) \\
& J_1(N)^{\text{new}} \arrow[ur, twoheadrightarrow] & \\
X_1(N) \arrow[uu, hookrightarrow] \arrow[rr, twoheadrightarrow, "\pi"] & & C \arrow[uu, hookrightarrow]
\end{tikzcd}$$

Then

$$\pi^* H^0(C,\Omega^1) \hookrightarrow S_2(N)^{\text{new}} \frac{dq}{q}$$

Let C be a new modular curve of level N

then $\exists f_1, \ldots, f_k \in \mathrm{New}_N := \{\text{normalized newforms on } S_2(N)\}$ such that

$$J_1(N)^{\text{new}} \overset{\mathbb{Q}}{\sim} \prod_{f \in \mathrm{New}_N / G_{\mathbb{Q}}} A_f \twoheadrightarrow J(C) \overset{\mathbb{Q}}{\sim} \prod_{i=1}^{k} A_{f_i},$$

Then

$$\pi^*(H^0(C,\Omega^1)) = \bigoplus_{i=1}^{k} S_2(A_{f_i}) \frac{dq}{q} = \bigoplus_{i=1}^{k} \langle {}^{\sigma} f_i(q) \mid \sigma \in \mathrm{Gal}(E_{f_i}/\mathbb{Q}) \rangle \frac{dq}{q}$$

Let C be a new modular curve of level N
then $\exists f_1, \dots, f_k \in \mathrm{New}_N := \{\text{normalized newforms on } S_2(N)\}$ such that

$$J_1(N)^{\text{new}} \overset{\mathbb{Q}}{\sim} \prod_{f \in \mathrm{New}_N / G_{\mathbb{Q}}} A_f \longrightarrow\!\!\!\!\rightarrow J(C) \overset{\mathbb{Q}}{\sim} \prod_{i=1}^{k} A_{f_i},$$

Then

$$\pi^*(H^0(C, \Omega^1)) = \bigoplus_{i=1}^{k} S_2(A_{f_i}) \frac{dq}{q} = \bigoplus_{i=1}^{k} \langle {}^{\sigma} f_i(q) \mid \sigma \in \mathrm{Gal}(E_{f_i}/\mathbb{Q}) \rangle \frac{dq}{q}$$

Let C be a new modular curve of level N
then $\exists f_1,\dots,f_k \in \mathrm{New}_N := \{\text{normalized newforms on } S_2(N)\}$ such that

$$J_1(N)^{\text{new}} \overset{\mathbb{Q}}{\sim} \prod_{f \in \mathrm{New}_N/G_{\mathbb{Q}}} A_f \longrightarrow\!\!\!\!\to J(C) \overset{\mathbb{Q}}{\sim} \prod_{i=1}^{k} A_{f_i}\,,$$

Then

$$\pi^*(H^0(C,\Omega^1)) = \bigoplus_{i=1}^{k} S_2(A_{f_i})\,\frac{dq}{q} = \bigoplus_{i=1}^{k} \langle {}^{\sigma} f_i(q) \mid \sigma \in \mathrm{Gal}(E_{f_i}/\mathbb{Q})\rangle\,\frac{dq}{q}$$

Let C be a new modular curve of level N
then $\exists f_1, \ldots, f_k \in \mathrm{New}_N := \{\text{normalized newforms on } S_2(N)\}$ such that

$$J_1(N)^{\text{new}} \overset{\mathbb{Q}}{\sim} \prod_{f \in \mathrm{New}_N / G_{\mathbb{Q}}} A_f \longrightarrow\!\!\!\!\!\rightarrow J(C) \overset{\mathbb{Q}}{\sim} \prod_{i=1}^{k} A_{f_i},$$

Then

$$\pi^*(H^0(C,\Omega^1)) = \bigoplus_{i=1}^{k} S_2(A_{f_i}) \frac{dq}{q} = \bigoplus_{i=1}^{k} \langle {}^{\sigma} f_i(q) \mid \sigma \in \mathrm{Gal}(E_{f_i}/\mathbb{Q}) \rangle \frac{dq}{q}$$

Let C be a new modular curve of level N
then $\exists f_1, \ldots, f_k \in \mathrm{New}_N := \{\text{normalized newforms on } S_2(N)\}$ such that

$$J_1(N)^{\text{new}} \overset{\mathbb{Q}}{\sim} \prod_{f \in \mathrm{New}_N / G_{\mathbb{Q}}} A_f \twoheadrightarrow J(C) \overset{\mathbb{Q}}{\sim} \prod_{i=1}^{k} A_{f_i} ,$$

Then

$$\pi^*(H^0(C, \Omega^1)) = \bigoplus_{i=1}^{k} S_2(A_{f_i}) \frac{dq}{q} = \bigoplus_{i=1}^{k} \langle {}^{\sigma} f_i(q) \mid \sigma \in \mathrm{Gal}(E_{f_i}/\mathbb{Q}) \rangle \frac{dq}{q}$$

Let C be a new modular curve of level N
then $\exists f_1, \ldots, f_k \in \mathrm{New}_N := \{\text{normalized newforms on } S_2(N)\}$ such that

$$J_1(N)^{\text{new}} \overset{\mathbb{Q}}{\sim} \prod_{f \in \mathrm{New}_N / G_{\mathbb{Q}}} A_f \longrightarrow\!\!\!\!\!\rightarrow J(C) \overset{\mathbb{Q}}{\sim} \prod_{i=1}^{k} A_{f_i},$$

Then

$$\pi^*(H^0(C,\Omega^1)) = \bigoplus_{i=1}^{k} S_2(A_{f_i}) \frac{dq}{q} = \bigoplus_{i=1}^{k} \langle {}^{\sigma} f_i(q) \mid \sigma \in \mathrm{Gal}(E_{f_i}/\mathbb{Q}) \rangle \frac{dq}{q}$$

Let C be a new modular curve of level N then $\exists f_1,\dots,f_k \in \mathrm{New}_N := \{\text{normalized newforms on } S_2(N)\}$ such that

$$J_1(N)^{\text{new}} \overset{\mathbb{Q}}{\sim} \prod_{f \in \mathrm{New}_N/G_{\mathbb{Q}}} A_f \twoheadrightarrow J(C) \overset{\mathbb{Q}}{\sim} \prod_{i=1}^{k} A_{f_i}\,,$$

Then

$$\pi^*(H^0(C,\Omega^1)) = \bigoplus_{i=1}^{k} S_2(A_{f_i})\,\frac{dq}{q} = \bigoplus_{i=1}^{k} \langle {}^{\sigma} f_i(q) \mid \sigma \in \mathrm{Gal}(E_{f_i}/\mathbb{Q})\rangle\,\frac{dq}{q}$$

Let C be a new modular curve of level N then $\exists f_1, \ldots, f_k \in \mathrm{New}_N := \{\text{normalized newforms on } S_2(N)\}$ such that

$$J_1(N)^{\text{new}} \stackrel{\mathbb{Q}}{\sim} \prod_{f \in \mathrm{New}_N / G_{\mathbb{Q}}} A_f \twoheadrightarrow J(C) \stackrel{\mathbb{Q}}{\sim} \prod_{i=1}^{k} A_{f_i}\,,$$

Then

$$\pi^*(H^0(C,\Omega^1)) = \bigoplus_{i=1}^{k} S_2(A_{f_i})\,\frac{dq}{q} = \bigoplus_{i=1}^{k} \langle {}^{\sigma} f_i(q) \mid \sigma \in \mathrm{Gal}(E_{f_i}/\mathbb{Q}) \rangle\,\frac{dq}{q}$$

Let C be a new modular curve of level N then $\exists f_1,\ldots,f_k \in \mathrm{New}_N := \{\text{normalized newforms on } S_2(N)\}$ such that

$$J_1(N)^{\mathrm{new}} \overset{\mathbb{Q}}{\sim} \prod_{f\in \mathrm{New}_N/G_{\mathbb{Q}}} A_f \longrightarrow\!\!\!\!\!\rightarrow J(C) \overset{\mathbb{Q}}{\sim} \prod_{i=1}^{k} A_{f_i}\,,$$

Then

$$\pi^*(H^0(C,\Omega^1)) = \bigoplus_{i=1}^{k} S_2(A_{f_i})\,\frac{dq}{q} = \bigoplus_{i=1}^{k} \langle {}^{\sigma} f_i(q) \mid \sigma \in \mathrm{Gal}(E_{f_i}/\mathbb{Q})\rangle\,\frac{dq}{q}$$

Let C be a new modular curve of level N then $\exists f_1,\dots,f_k \in \mathrm{New}_N := \{\text{normalized newforms on } S_2(N)\}$ such that

$$J_1(N)^{\text{new}} \overset{\mathbb{Q}}{\sim} \prod_{f\in \mathrm{New}_N/G_{\mathbb{Q}}} A_f \longrightarrow\!\!\!\!\!\to J(C) \overset{\mathbb{Q}}{\sim} \prod_{i=1}^{k} A_{f_i}\,,$$

Then

$$\pi^*(H^0(C,\Omega^1)) = \bigoplus_{i=1}^{k} S_2(A_{f_i})\,\frac{dq}{q} = \bigoplus_{i=1}^{k} \langle {}^{\sigma} f_i(q) \mid \sigma \in \mathrm{Gal}(E_{f_i}/\mathbb{Q})\rangle\,\frac{dq}{q}$$

Let C be a new modular curve of level N
then $\exists f_1, \ldots, f_k \in \mathrm{New}_N := \{\text{normalized newforms on } S_2(N)\}$ such that

$$J_1(N)^{\text{new}} \overset{\mathbb{Q}}{\sim} \prod_{f \in \mathrm{New}_N / G_{\mathbb{Q}}} A_f \longrightarrow\!\!\!\!\rightarrow J(C) \overset{\mathbb{Q}}{\sim} \prod_{i=1}^{k} A_{f_i},$$

Then

$$\pi^*(H^0(C, \Omega^1)) = \bigoplus_{i=1}^{k} S_2(A_{f_i}) \frac{dq}{q} = \bigoplus_{i=1}^{k} \langle {}^{\sigma} f_i(q) \mid \sigma \in \mathrm{Gal}(E_{f_i}/\mathbb{Q}) \rangle \frac{dq}{q}$$

Let C be a new modular curve of level N
then $\exists f_1, \ldots, f_k \in \mathrm{New}_N := \{\text{normalized newforms on } S_2(N)\}$ such that

$$J_1(N)^{\text{new}} \overset{\mathbb{Q}}{\sim} \prod_{f \in \mathrm{New}_N / G_{\mathbb{Q}}} A_f \twoheadrightarrow J(C) \overset{\mathbb{Q}}{\sim} \prod_{i=1}^{k} A_{f_i},$$

Then

$$\pi^*(H^0(C,\Omega^1)) = \bigoplus_{i=1}^{k} S_2(A_{f_i}) \frac{dq}{q} = \bigoplus_{i=1}^{k} \langle {}^{\sigma} f_i(q) \mid \sigma \in \mathrm{Gal}(E_{f_i}/\mathbb{Q}) \rangle \frac{dq}{q}$$

Let C be a new modular curve of level N
then $\exists f_1, \ldots, f_k \in \mathrm{New}_N := \{\text{normalized newforms on } S_2(N)\}$ such that

$$J_1(N)^{\mathrm{new}} \overset{\mathbb{Q}}{\sim} \prod_{f \in \mathrm{New}_N / G_{\mathbb{Q}}} A_f \longrightarrow\!\!\!\!\!\rightarrow J(C) \overset{\mathbb{Q}}{\sim} \prod_{i=1}^{k} A_{f_i}\,,$$

Then

$$\pi^*(H^0(C, \Omega^1)) = \bigoplus_{i=1}^{k} S_2(A_{f_i})\,\frac{dq}{q} = \bigoplus_{i=1}^{k} \langle {}^{\sigma} f_i(q) \mid \sigma \in \mathrm{Gal}(E_{f_i}/\mathbb{Q}) \rangle\,\frac{dq}{q}$$

Let C be a new modular curve of level N
then $\exists f_1,\ldots,f_k \in \mathrm{New}_N := \{\text{normalized newforms on } S_2(N)\}$ such that

$$J_1(N)^{\text{new}} \overset{\mathbb{Q}}{\sim} \prod_{f\in \mathrm{New}_N/G_{\mathbb{Q}}} A_f \longrightarrow\!\!\!\!\rightarrow J(C) \overset{\mathbb{Q}}{\sim} \prod_{i=1}^{k} A_{f_i}\,,$$

Then

$$\pi^*(H^0(C,\Omega^1)) = \bigoplus_{i=1}^{k} S_2(A_{f_i})\,\frac{dq}{q} = \bigoplus_{i=1}^{k} \langle {}^{\sigma} f_i(q) \mid \sigma \in \mathrm{Gal}(E_{f_i}/\mathbb{Q})\rangle\,\frac{dq}{q}$$

Let C be a new modular curve of level N
then $\exists f_1,\ldots,f_k \in \mathrm{New}_N := \{\text{normalized newforms on } S_2(N)\}$ such that

$$J_1(N)^{\text{new}} \overset{\mathbb{Q}}{\sim} \prod_{f\in \mathrm{New}_N/G_{\mathbb{Q}}} A_f \longrightarrow\!\!\!\!\!\rightarrow J(C) \overset{\mathbb{Q}}{\sim} \prod_{i=1}^{k} A_{f_i}\,,$$

Then

$$\pi^*(H^0(C,\Omega^1)) = \bigoplus_{i=1}^{k} S_2(A_{f_i})\,\frac{dq}{q} = \bigoplus_{i=1}^{k} \langle {}^{\sigma}f_i(q) \mid \sigma \in \mathrm{Gal}(E_{f_i}/\mathbb{Q})\rangle\,\frac{dq}{q}$$

Equations for Modular Curves

Fact

If $C_{/\mathbb{Q}}$ is modular of level N, then $C(\mathbb{Q}) \neq \emptyset$, since $\pi_{/\mathbb{Q}}$ and $i\infty \in X_1(N)(\mathbb{Q})$.

Fact

If $C_{/\mathbb{Q}}$ is modular of level N, then $C(\mathbb{Q}) \neq \emptyset$,
since $\pi_{/\mathbb{Q}}$ and $i\infty \in X_1(N)(\mathbb{Q})$.

Fact

If $C_{/\mathbb{Q}}$ is modular of level N, then $C(\mathbb{Q}) \neq \emptyset$,
since $\pi_{/\mathbb{Q}}$ and $i\infty \in X_1(N)(\mathbb{Q})$.

Notation

Let $g \in \mathbb{Z}$ such that $g \geq 0$, we denote by

$$\mathcal{MC}_g = \{\text{modular curves of genus } g\}_{/\underset{\mathbb{Q}}{\simeq}},$$

$$\mathcal{MC}_g^{\text{new}} = \{[C] \in \mathcal{MC}_g \mid C \text{ is new}\}.$$

Notation

Let $g \in \mathbb{Z}$ such that $g \geq 0$, we denote by

$$\mathcal{MC}_g = \{\text{modular curves of genus } g\}_{/\overset{\mathbb{Q}}{\simeq}},$$

$$\mathcal{MC}_g^{\text{new}} = \{[C] \in \mathcal{MC}_g \mid C \text{ is new}\}.$$

Notation

Let $g \in \mathbb{Z}$ such that $g \geq 0$, we denote by

$$\begin{aligned} \mathcal{MC}_g &= \{\text{modular curves of genus } g\}_{/\overset{\mathbb{Q}}{\simeq}}, \\ \mathcal{MC}_g^{\text{new}} &= \{[C] \in \mathcal{MC}_g \mid C \text{ is new}\}. \end{aligned}$$

$g = 0$

$\mathcal{MC}_0 = \mathcal{MC}_0^{\text{new}} = \{X_1(1)\}.$

$\#\mathcal{MC}_0 = \#\mathcal{MC}_0^{\text{new}} = 1.$

$g = 1$

$\mathcal{MC}_1 = \mathcal{MC}_1^{\text{new}} = \{\text{elliptic curves defined over } \mathbb{Q}\}_{/\overset{\mathbb{Q}}{\simeq}}.$

$\#\mathcal{MC}_1 = \#\mathcal{MC}_1^{\text{new}} = \infty.$

$g = 0$

$\mathcal{MC}_0 = \mathcal{MC}_0^{\text{new}} = \{X_1(1)\}.$

$\#\mathcal{MC}_0 = \#\mathcal{MC}_0^{\text{new}} = 1.$

$g = 1$

$\mathcal{MC}_1 = \mathcal{MC}_1^{\text{new}} = \{\text{elliptic curves defined over } \mathbb{Q}\}_{/\overset{\mathbb{Q}}{\simeq}}.$

$\#\mathcal{MC}_1 = \#\mathcal{MC}_1^{\text{new}} = \infty.$

$g = 0$

$\mathcal{MC}_0 = \mathcal{MC}_0^{\text{new}} = \{X_1(1)\}.$

$\#\mathcal{MC}_0 = \#\mathcal{MC}_0^{\text{new}} = 1.$

$g = 1$

$\mathcal{MC}_1 = \mathcal{MC}_1^{\text{new}} = \{\text{elliptic curves defined over } \mathbb{Q}\}_{/\overset{\mathbb{Q}}{\simeq}}.$

$\#\mathcal{MC}_1 = \#\mathcal{MC}_1^{\text{new}} = \infty.$

$g = 0$

$$\mathcal{MC}_0 = \mathcal{MC}_0^{\text{new}} = \{X_1(1)\}.$$

$$\#\mathcal{MC}_0 = \#\mathcal{MC}_0^{\text{new}} = 1.$$

$g = 1$

$$\mathcal{MC}_1 = \mathcal{MC}_1^{\text{new}} = \{\text{elliptic curves defined over } \mathbb{Q}\}_{/\overset{\mathbb{Q}}{\simeq}}.$$

$$\#\mathcal{MC}_1 = \#\mathcal{MC}_1^{\text{new}} = \infty.$$

$g = 0$

$$\mathcal{MC}_0 = \mathcal{MC}_0^{\text{new}} = \{X_1(1)\}.$$

$$\#\mathcal{MC}_0 = \#\mathcal{MC}_0^{\text{new}} = 1.$$

$g = 1$

$$\mathcal{MC}_1 = \mathcal{MC}_1^{\text{new}} = \{\text{elliptic curves defined over } \mathbb{Q}\}_{/\overset{\mathbb{Q}}{\simeq}}.$$

$$\#\mathcal{MC}_1 = \#\mathcal{MC}_1^{\text{new}} = \infty.$$

$g = 0$

$$\mathcal{MC}_0 = \mathcal{MC}_0^{\text{new}} = \{X_1(1)\}.$$

$$\#\mathcal{MC}_0 = \#\mathcal{MC}_0^{\text{new}} = 1.$$

$g = 1$

$$\mathcal{MC}_1 = \mathcal{MC}_1^{\text{new}} = \{\text{elliptic curves defined over } \mathbb{Q}\}_{/\overset{\mathbb{Q}}{\simeq}}.$$

$$\#\mathcal{MC}_1 = \#\mathcal{MC}_1^{\text{new}} = \infty.$$

$g = 0$

$$\mathcal{MC}_0 = \mathcal{MC}_0^{\text{new}} = \{X_1(1)\}.$$

$$\#\mathcal{MC}_0 = \#\mathcal{MC}_0^{\text{new}} = 1.$$

$g = 1$

$$\mathcal{MC}_1 = \mathcal{MC}_1^{\text{new}} = \{\text{elliptic curves defined over } \mathbb{Q}\}_{/\overset{\mathbb{Q}}{\simeq}}.$$

$$\#\mathcal{MC}_1 = \#\mathcal{MC}_1^{\text{new}} = \infty.$$

$g = 0$

$$\mathcal{MC}_0 = \mathcal{MC}_0^{\text{new}} = \{X_1(1)\}.$$

$$\#\mathcal{MC}_0 = \#\mathcal{MC}_0^{\text{new}} = 1.$$

$g = 1$

$$\mathcal{MC}_1 = \mathcal{MC}_1^{\text{new}} = \{\text{elliptic curves defined over } \mathbb{Q}\}_{/\underset{\mathbb{Q}}{\simeq}}.$$

$$\#\mathcal{MC}_1 = \#\mathcal{MC}_1^{\text{new}} = \infty.$$

$g = 0$

$$\mathcal{MC}_0 = \mathcal{MC}_0^{\text{new}} = \{X_1(1)\}.$$

$$\#\mathcal{MC}_0 = \#\mathcal{MC}_0^{\text{new}} = 1.$$

$g = 1$

$$\mathcal{MC}_1 = \mathcal{MC}_1^{\text{new}} = \{\text{elliptic curves defined over } \mathbb{Q}\}_{/\overset{\mathbb{Q}}{\simeq}}.$$

$$\#\mathcal{MC}_1 = \#\mathcal{MC}_1^{\text{new}} = \infty.$$

$g = 0$

$$\mathcal{MC}_0 = \mathcal{MC}_0^{\text{new}} = \{X_1(1)\}.$$

$$\#\mathcal{MC}_0 = \#\mathcal{MC}_0^{\text{new}} = 1.$$

$g = 1$

$$\mathcal{MC}_1 = \mathcal{MC}_1^{\text{new}} = \{\text{elliptic curves defined over } \mathbb{Q}\}_{/\overset{\mathbb{Q}}{\simeq}}.$$

$$\#\mathcal{MC}_1 = \#\mathcal{MC}_1^{\text{new}} = \infty.$$

$g = 2$

Theorem

$\#\mathcal{MC}_2^{new} < \infty$

E. González-Jiménez and J. González,
Modular curves of genus 2,
Math. Comp. **72, (241** (2003), 397–418.

$g = 2$

Theorem

$\#\mathcal{M}\mathcal{C}_2^{new} < \infty$

E. González-Jiménez and J. González,
Modular curves of genus 2,
Math. Comp. **72, (241** (2003), 397–418.

Theorem

$\#\mathcal{MC}_2^{new} < \infty$

Moreover,

- 96 *have* $\overline{\mathbb{Q}}$*-simple jacobian*
- 53 *have* $\mathbb{Q}$*-simple but no* $\overline{\mathbb{Q}}$*-simple jacobian*

E. González-Jiménez and J. González,
Modular curves of genus 2,
Math. Comp. **72, (241)** (2003), 397–418.

Theorem

$\#\mathcal{MC}_2^{new} < \infty$

Moreover,

- 96 *have* $\overline{\mathbb{Q}}$*-simple jacobian*
- 53 *have* $\mathbb{Q}$*-simple but no* $\overline{\mathbb{Q}}$*-simple jacobian*

E. González-Jiménez and J. González,
Modular curves of genus 2,
Math. Comp. **72, (241)** (2003), 397–418.

Theorem

$\#\mathcal{MC}_2^{new} < \infty$

Moreover,

- 96 *have* $\overline{\mathbb{Q}}$*-simple jacobian*
- 53 *have* $\mathbb{Q}$*-simple but no* $\overline{\mathbb{Q}}$*-simple jacobian*

E. González-Jiménez and J. González,
Modular curves of genus 2,
Math. Comp. **72, (241)** (2003), 397–418.

Theorem

$\#\mathcal{MC}_2^{new} < \infty$

Moreover,

- 96 *have* $\overline{\mathbb{Q}}$*-simple jacobian*
- 53 *have* $\mathbb{Q}$*-simple but no* $\overline{\mathbb{Q}}$*-simple jacobian*

E. González-Jiménez and J. González,
Modular curves of genus 2,
Math. Comp. **72, (241)** (2003), 397–418.

Theorem

$\#\mathcal{M}\mathcal{C}_2^{new} = 213$

Moreover,

- 96 *have* $\overline{\mathbb{Q}}$*-simple jacobian,*
- 53 *have* $\mathbb{Q}$*-simple but no* $\overline{\mathbb{Q}}$*-simple jacobian,*
- 64 *have no* $\mathbb{Q}$*-simple jacobian.*

E. González-Jiménez and J. González,
Modular curves of genus 2,
Math. Comp. **72, (241)** (2003), 397–418.

M.H. Baker, E. González-Jiménez, J. González, and B. Poonen.,
Finiteness results for modular curves of genus at least 2,
Amer. J. Math. **127** (2005), no. 6, 1325–1387.

Theorem

$\#\mathcal{M}\mathcal{C}_2^{new} = 213$

Moreover,

- 96 *have* $\overline{\mathbb{Q}}$*-simple jacobian,*
- 53 *have* $\mathbb{Q}$*-simple but no* $\overline{\mathbb{Q}}$*-simple jacobian,*
- 64 *have no* $\mathbb{Q}$*-simple jacobian.*

E. González-Jiménez and J. González,
Modular curves of genus 2,
Math. Comp. **72, (241)** (2003), 397–418.

M.H. Baker, E. González-Jiménez, J. González, and B. Poonen.,
Finiteness results for modular curves of genus at least 2,
Amer. J. Math. **127** (2005), no. 6, 1325–1387.

Theorem

$\#\mathcal{MC}_2^{new} = 213$

Moreover,

- 96 *have* $\overline{\mathbb{Q}}$*-simple jacobian,*
- 53 *have* $\mathbb{Q}$*-simple but no* $\overline{\mathbb{Q}}$*-simple jacobian,*
- 64 *have no* $\mathbb{Q}$*-simple jacobian.*

E. González-Jiménez and J. González,
Modular curves of genus 2,
Math. Comp. **72, (241)** (2003), 397–418.

M.H. Baker, E. González-Jiménez, J. González, and B. Poonen.,
Finiteness results for modular curves of genus at least 2,
Amer. J. Math. **127** (2005), no. 6, 1325–1387.

Theorem

$\#\mathcal{M}\mathcal{C}_2^{new} = 213$

Moreover,

- 96 *have* $\overline{\mathbb{Q}}$*-simple jacobian,*
- 53 *have* $\mathbb{Q}$*-simple but no* $\overline{\mathbb{Q}}$*-simple jacobian,*
- 64 *have no* $\mathbb{Q}$*-simple jacobian.*

E. González-Jiménez and J. González,
Modular curves of genus 2,
Math. Comp. **72, (241)** (2003), 397–418.

M.H. Baker, E. González-Jiménez, J. González, and B. Poonen.,
Finiteness results for modular curves of genus at least 2,
Amer. J. Math. **127** (2005), no. 6, 1325–1387.

Theorem

$\#\mathcal{MC}_2^{new} = 213$

Moreover,

- 96 *have* $\overline{\mathbb{Q}}$*-simple jacobian,*
- 53 *have* $\mathbb{Q}$*-simple but no* $\overline{\mathbb{Q}}$*-simple jacobian,*
- 64 *have no* $\mathbb{Q}$*-simple jacobian.*

E. González-Jiménez and J. González,
Modular curves of genus 2,
Math. Comp. **72, (241)** (2003), 397–418.

M.H. Baker, E. González-Jiménez, J. González, and B. Poonen.,
Finiteness results for modular curves of genus at least 2,
Amer. J. Math. **127** (2005), no. 6, 1325–1387.

Theorem

$\#\mathcal{MC}_2^{new} = 213$

Moreover,

- 96 *have* $\overline{\mathbb{Q}}$*-simple jacobian,*
- 53 *have* $\mathbb{Q}$*-simple but no* $\overline{\mathbb{Q}}$*-simple jacobian,*
- 64 *have no* $\mathbb{Q}$*-simple jacobian.*

E. González-Jiménez and J. González,
Modular curves of genus 2,
Math. Comp. **72, (241)** (2003), 397–418.

M.H. Baker, E. González-Jiménez, J. González, and B. Poonen.,
Finiteness results for modular curves of genus at least 2,
Amer. J. Math. **127** (2005), no. 6, 1325–1387.

C	:	$y^2 = F(x)$
$X_1(13)$	:	$y^2 = x^6 - 2x^5 + x^4 - 2x^3 + 6x^2 - 4x + 1$
$X_1(16)$	:	$y^2 = x^6 + 2x^5 - x^4 - x^2 - 2x + 1$
$X_1(18)$	:	$y^2 = x^6 - 4x^5 + 10x^4 - 10x^3 + 5x^2 - 2x + 1$
$X_0(23)$	:	$y^2 = x^6 - 8x^5 + 2x^4 + 2x^3 - 11x^2 + 10x - 7$
$X_0(26)$	:	$y^2 = x^6 - 8x^5 + 8x^4 - 18x^3 + 8x^2 - 8x + 1$
$C_{28A_{\{0,2\}}}$	:	$y^2 = x^5 - 9x^4 + 13x^3 - 4x^2 - x$
$\vdots$		$\vdots$
		$\boxed{\text{Total: 213 curves}}$
$\vdots$		$\vdots$
C_{7424A}	:	$y^2 = x^5 + 10x^3 - 4x$
C_{7424B}	:	$y^2 = x^5 - 10x^3 - 4x$
C_{7664A}	:	$y^2 = x^5 - 3x^4 - 7x^3 - 5x^2 - 2x - 1$

C	:	$y^2 = F(x)$
$X_1(13)$	:	$y^2 = x^6 - 2x^5 + x^4 - 2x^3 + 6x^2 - 4x + 1$
$X_1(16)$	:	$y^2 = x^6 + 2x^5 - x^4 - x^2 - 2x + 1$
$X_1(18)$	:	$y^2 = x^6 - 4x^5 + 10x^4 - 10x^3 + 5x^2 - 2x + 1$
$X_0(23)$	:	$y^2 = x^6 - 8x^5 + 2x^4 + 2x^3 - 11x^2 + 10x - 7$
$X_0(26)$	:	$y^2 = x^6 - 8x^5 + 8x^4 - 18x^3 + 8x^2 - 8x + 1$
$C_{28A_{\{0,2\}}}$	:	$y^2 = x^5 - 9x^4 + 13x^3 - 4x^2 - x$
$\vdots$		$\vdots$
		$\boxed{\text{Total: 213 curves}}$
$\vdots$		$\vdots$
C_{7424A}	:	$y^2 = x^5 + 10x^3 - 4x$
C_{7424B}	:	$y^2 = x^5 - 10x^3 - 4x$
C_{7664A}	:	$y^2 = x^5 - 3x^4 - 7x^3 - 5x^2 - 2x - 1$

C	:	$y^2 = F(x)$
$X_1(13)$	:	$y^2 = x^6 - 2x^5 + x^4 - 2x^3 + 6x^2 - 4x + 1$
$X_1(16)$	:	$y^2 = x^6 + 2x^5 - x^4 - x^2 - 2x + 1$
$X_1(18)$	:	$y^2 = x^6 - 4x^5 + 10x^4 - 10x^3 + 5x^2 - 2x + 1$
$X_0(23)$	:	$y^2 = x^6 - 8x^5 + 2x^4 + 2x^3 - 11x^2 + 10x - 7$
$X_0(26)$	:	$y^2 = x^6 - 8x^5 + 8x^4 - 18x^3 + 8x^2 - 8x + 1$
$C_{28A_{\{0,2\}}}$	:	$y^2 = x^5 - 9x^4 + 13x^3 - 4x^2 - x$
$\vdots$		$\vdots$
		$\boxed{\text{Total: 213 curves}}$
$\vdots$		$\vdots$
C_{7424A}	:	$y^2 = x^5 + 10x^3 - 4x$
C_{7424B}	:	$y^2 = x^5 - 10x^3 - 4x$
C_{7664A}	:	$y^2 = x^5 - 3x^4 - 7x^3 - 5x^2 - 2x - 1$

C	:	$y^2 = F(x)$
$X_1(13)$	:	$y^2 = x^6 - 2x^5 + x^4 - 2x^3 + 6x^2 - 4x + 1$
$X_1(16)$	:	$y^2 = x^6 + 2x^5 - x^4 - x^2 - 2x + 1$
$X_1(18)$	:	$y^2 = x^6 - 4x^5 + 10x^4 - 10x^3 + 5x^2 - 2x + 1$
$X_0(23)$	:	$y^2 = x^6 - 8x^5 + 2x^4 + 2x^3 - 11x^2 + 10x - 7$
$X_0(26)$	:	$y^2 = x^6 - 8x^5 + 8x^4 - 18x^3 + 8x^2 - 8x + 1$
$C_{28A_{\{0,2\}}}$	:	$y^2 = x^5 - 9x^4 + 13x^3 - 4x^2 - x$
$\vdots$		$\vdots$
		$\boxed{\text{Total: 213 curves}}$
$\vdots$		$\vdots$
C_{7424A}	:	$y^2 = x^5 + 10x^3 - 4x$
C_{7424B}	:	$y^2 = x^5 - 10x^3 - 4x$
C_{7664A}	:	$y^2 = x^5 - 3x^4 - 7x^3 - 5x^2 - 2x - 1$

C	:	$y^2 = F(x)$
$X_1(13)$	:	$y^2 = x^6 - 2x^5 + x^4 - 2x^3 + 6x^2 - 4x + 1$
$X_1(16)$	:	$y^2 = x^6 + 2x^5 - x^4 - x^2 - 2x + 1$
$X_1(18)$	:	$y^2 = x^6 - 4x^5 + 10x^4 - 10x^3 + 5x^2 - 2x + 1$
$X_0(23)$	:	$y^2 = x^6 - 8x^5 + 2x^4 + 2x^3 - 11x^2 + 10x - 7$
$X_0(26)$	:	$y^2 = x^6 - 8x^5 + 8x^4 - 18x^3 + 8x^2 - 8x + 1$
$C_{28A_{\{0,2\}}}$	:	$y^2 = x^5 - 9x^4 + 13x^3 - 4x^2 - x$
$\vdots$		$\vdots$
		Total: 213 curves
$\vdots$		$\vdots$
C_{7424A}	:	$y^2 = x^5 + 10x^3 - 4x$
C_{7424B}	:	$y^2 = x^5 - 10x^3 - 4x$
C_{7664A}	:	$y^2 = x^5 - 3x^4 - 7x^3 - 5x^2 - 2x - 1$

C	:	$y^2 = F(x)$
$X_1(13)$	:	$y^2 = x^6 - 2x^5 + x^4 - 2x^3 + 6x^2 - 4x + 1$
$X_1(16)$	:	$y^2 = x^6 + 2x^5 - x^4 - x^2 - 2x + 1$
$X_1(18)$	:	$y^2 = x^6 - 4x^5 + 10x^4 - 10x^3 + 5x^2 - 2x + 1$
$X_0(23)$	:	$y^2 = x^6 - 8x^5 + 2x^4 + 2x^3 - 11x^2 + 10x - 7$
$X_0(26)$	:	$y^2 = x^6 - 8x^5 + 8x^4 - 18x^3 + 8x^2 - 8x + 1$
$C_{28A_{\{0,2\}}}$	:	$y^2 = x^5 - 9x^4 + 13x^3 - 4x^2 - x$
$\vdots$		$\vdots$
		$\boxed{\text{Total: 213 curves}}$
$\vdots$		$\vdots$
C_{7424A}	:	$y^2 = x^5 + 10x^3 - 4x$
C_{7424B}	:	$y^2 = x^5 - 10x^3 - 4x$
C_{7664A}	:	$y^2 = x^5 - 3x^4 - 7x^3 - 5x^2 - 2x - 1$

C	:	$y^2 = F(x)$
$X_1(13)$	:	$y^2 = x^6 - 2x^5 + x^4 - 2x^3 + 6x^2 - 4x + 1$
$X_1(16)$	:	$y^2 = x^6 + 2x^5 - x^4 - x^2 - 2x + 1$
$X_1(18)$	:	$y^2 = x^6 - 4x^5 + 10x^4 - 10x^3 + 5x^2 - 2x + 1$
$X_0(23)$	:	$y^2 = x^6 - 8x^5 + 2x^4 + 2x^3 - 11x^2 + 10x - 7$
$X_0(26)$	:	$y^2 = x^6 - 8x^5 + 8x^4 - 18x^3 + 8x^2 - 8x + 1$
$C_{28A_{\{0,2\}}}$	:	$y^2 = x^5 - 9x^4 + 13x^3 - 4x^2 - x$
$\vdots$		$\vdots$
		$\boxed{\text{Total: 213 curves}}$
$\vdots$		$\vdots$
C_{7424A}	:	$y^2 = x^5 + 10x^3 - 4x$
C_{7424B}	:	$y^2 = x^5 - 10x^3 - 4x$
C_{7664A}	:	$y^2 = x^5 - 3x^4 - 7x^3 - 5x^2 - 2x - 1$

C	:	$y^2 = F(x)$
$X_1(13)$	:	$y^2 = x^6 - 2x^5 + x^4 - 2x^3 + 6x^2 - 4x + 1$
$X_1(16)$	:	$y^2 = x^6 + 2x^5 - x^4 - x^2 - 2x + 1$
$X_1(18)$	:	$y^2 = x^6 - 4x^5 + 10x^4 - 10x^3 + 5x^2 - 2x + 1$
$X_0(23)$	:	$y^2 = x^6 - 8x^5 + 2x^4 + 2x^3 - 11x^2 + 10x - 7$
$X_0(26)$	:	$y^2 = x^6 - 8x^5 + 8x^4 - 18x^3 + 8x^2 - 8x + 1$
$C_{28A_{\{0,2\}}}$	:	$y^2 = x^5 - 9x^4 + 13x^3 - 4x^2 - x$
$\vdots$		$\vdots$
		$\boxed{\text{Total: 213 curves}}$
$\vdots$		$\vdots$
C_{7424A}	:	$y^2 = x^5 + 10x^3 - 4x$
C_{7424B}	:	$y^2 = x^5 - 10x^3 - 4x$
C_{7664A}	:	$y^2 = x^5 - 3x^4 - 7x^3 - 5x^2 - 2x - 1$

C	:	$y^2 = F(x)$
$X_1(13)$	:	$y^2 = x^6 - 2x^5 + x^4 - 2x^3 + 6x^2 - 4x + 1$
$X_1(16)$	:	$y^2 = x^6 + 2x^5 - x^4 - x^2 - 2x + 1$
$X_1(18)$	:	$y^2 = x^6 - 4x^5 + 10x^4 - 10x^3 + 5x^2 - 2x + 1$
$X_0(23)$	:	$y^2 = x^6 - 8x^5 + 2x^4 + 2x^3 - 11x^2 + 10x - 7$
$X_0(26)$	:	$y^2 = x^6 - 8x^5 + 8x^4 - 18x^3 + 8x^2 - 8x + 1$
$C_{28A_{\{0,2\}}}$	:	$y^2 = x^5 - 9x^4 + 13x^3 - 4x^2 - x$
$\vdots$		$\vdots$
		$\boxed{\text{Total: 213 curves}}$
$\vdots$		$\vdots$
C_{7424A}	:	$y^2 = x^5 + 10x^3 - 4x$
C_{7424B}	:	$y^2 = x^5 - 10x^3 - 4x$
C_{7664A}	:	$y^2 = x^5 - 3x^4 - 7x^3 - 5x^2 - 2x - 1$

C	:	$y^2 = F(x)$
$X_1(13)$	:	$y^2=x^6-2x^5+x^4-2x^3+6x^2-4x+1$
$X_1(16)$	:	$y^2=x^6+2x^5-x^4-x^2-2x+1$
$X_1(18)$	:	$y^2=x^6-4x^5+10x^4-10x^3+5x^2-2x+1$
$X_0(23)$	:	$y^2=x^6-8x^5+2x^4+2x^3-11x^2+10x-7$
$X_0(26)$	:	$y^2=x^6-8x^5+8x^4-18x^3+8x^2-8x+1$
$C_{28A_{\{0,2\}}}$	:	$y^2=x^5-9x^4+13x^3-4x^2-x$
$\vdots$		$\vdots$
		$\boxed{\text{Total: 213 curves}}$
$\vdots$		$\vdots$
C_{7424A}	:	$y^2=x^5+10x^3-4x$
C_{7424B}	:	$y^2=x^5-10x^3-4x$
C_{7664A}	:	$y^2=x^5-3x^4-7x^3-5x^2-2x-1$

C	:	$y^2 = F(x)$
$X_1(13)$	:	$y^2 = x^6 - 2x^5 + x^4 - 2x^3 + 6x^2 - 4x + 1$
$X_1(16)$	:	$y^2 = x^6 + 2x^5 - x^4 - x^2 - 2x + 1$
$X_1(18)$	:	$y^2 = x^6 - 4x^5 + 10x^4 - 10x^3 + 5x^2 - 2x + 1$
$X_0(23)$	:	$y^2 = x^6 - 8x^5 + 2x^4 + 2x^3 - 11x^2 + 10x - 7$
$X_0(26)$	:	$y^2 = x^6 - 8x^5 + 8x^4 - 18x^3 + 8x^2 - 8x + 1$
$C_{28A_{\{0,2\}}}$	:	$y^2 = x^5 - 9x^4 + 13x^3 - 4x^2 - x$
$\vdots$		$\vdots$
		Total: 213 curves
$\vdots$		$\vdots$
C_{7424A}	:	$y^2 = x^5 + 10x^3 - 4x$
C_{7424B}	:	$y^2 = x^5 - 10x^3 - 4x$
C_{7664A}	:	$y^2 = x^5 - 3x^4 - 7x^3 - 5x^2 - 2x - 1$

$$
\begin{array}{lcl}
C & : & y^2 \;=\; F(x) \\
X_1(13) & : & y^2 = x^6 - 2x^5 + x^4 - 2x^3 + 6x^2 - 4x + 1 \\
X_1(16) & : & y^2 = x^6 + 2x^5 - x^4 - x^2 - 2x + 1 \\
X_1(18) & : & y^2 = x^6 - 4x^5 + 10x^4 - 10x^3 + 5x^2 - 2x + 1 \\
X_0(23) & : & y^2 = x^6 - 8x^5 + 2x^4 + 2x^3 - 11x^2 + 10x - 7 \\
X_0(26) & : & y^2 = x^6 - 8x^5 + 8x^4 - 18x^3 + 8x^2 - 8x + 1 \\
C_{28A_{\{0,2\}}} & : & y^2 = x^5 - 9x^4 + 13x^3 - 4x^2 - x \\
\vdots & & \qquad\qquad\qquad \vdots \\
 & & \boxed{\text{Total: 213 curves}} \\
\vdots & & \qquad\qquad\qquad \vdots \\
C_{7424A} & : & y^2 = x^5 + 10x^3 - 4x \\
C_{7424B} & : & y^2 = x^5 - 10x^3 - 4x \\
C_{7664A} & : & y^2 = x^5 - 3x^4 - 7x^3 - 5x^2 - 2x - 1
\end{array}
$$

C	:	$y^2 = F(x)$
$X_1(13)$	:	$y^2 = x^6 - 2x^5 + x^4 - 2x^3 + 6x^2 - 4x + 1$
$X_1(16)$	:	$y^2 = x^6 + 2x^5 - x^4 - x^2 - 2x + 1$
$X_1(18)$	:	$y^2 = x^6 - 4x^5 + 10x^4 - 10x^3 + 5x^2 - 2x + 1$
$X_0(23)$	:	$y^2 = x^6 - 8x^5 + 2x^4 + 2x^3 - 11x^2 + 10x - 7$
$X_0(26)$	:	$y^2 = x^6 - 8x^5 + 8x^4 - 18x^3 + 8x^2 - 8x + 1$
$C_{28A_{\{0,2\}}}$	:	$y^2 = x^5 - 9x^4 + 13x^3 - 4x^2 - x$
$\vdots$		$\vdots$
		$\boxed{\text{Total: 213 curves}}$
$\vdots$		$\vdots$
C_{7424A}	:	$y^2 = x^5 + 10x^3 - 4x$
C_{7424B}	:	$y^2 = x^5 - 10x^3 - 4x$
C_{7664A}	:	$y^2 = x^5 - 3x^4 - 7x^3 - 5x^2 - 2x - 1$

$g \geq 2$

Theorem

Let $g \geq 2$ be an integer. Then $\mathcal{MC}_g^{new}$ is finite and computable.

M.H. Baker, E. González-Jiménez, J. González, and B. Poonen., *Finiteness results for modular curves of genus at least 2*, Amer. J. Math. **127** (2005), no. 6, 1325–1387.

$g \geq 2$

Theorem

Let $g \geq 2$ be an integer. Then $\mathcal{M}\mathcal{C}_g^{new}$ is finite and computable.

M.H. Baker, E. González-Jiménez, J. González, and B. Poonen.,
Finiteness results for modular curves of genus at least 2,
Amer. J. Math. **127** (2005), no. 6, 1325–1387.

Notation

Let $G, g \geq 2$ be positive integers, we denote by

$$\mathcal{MC}_g^{\text{new}}(G) = \{[C] \in \mathcal{MC}_g^{\text{new}} \mid C \text{ has gonality } G\}.$$

And with more generality,

$$\mathcal{MC}^{\text{new}}(G) = \bigcup_{g \geq 2} \mathcal{MC}_g^{\text{new}}(G).$$

Theorem

Let $G \geq 2$ be an integer. Then $\mathcal{MC}^{new}(G)$ is finite and computable.

M.H. Baker, E. González-Jiménez, J. González, and B. Poonen., *Finiteness results for modular curves of genus at least 2*, Amer. J. Math. **127** (2005), no. 6, 1325–1387.

Notation

Let $G, g \geq 2$ be positive integers, we denote by

$$\mathcal{MC}_g^{\text{new}}(G) = \{[C] \in \mathcal{MC}_g^{\text{new}} \mid C \text{ has gonality } G\}.$$

And with more generality,

$$\mathcal{MC}^{\text{new}}(G) = \bigcup_{g \geq 2} \mathcal{MC}_g^{\text{new}}(G).$$

Theorem

Let $G \geq 2$ be an integer. Then $\mathcal{MC}^{new}(G)$ is finite and computable.

M.H. Baker, E. González-Jiménez, J. González, and B. Poonen., *Finiteness results for modular curves of genus at least 2*, Amer. J. Math. **127** (2005), no. 6, 1325–1387.

Notation

Let $G, g \geq 2$ be positive integers, we denote by

$$\mathcal{MC}_g^{\text{new}}(G) = \{[C] \in \mathcal{MC}_g^{\text{new}} \mid C \text{ has gonality } G\}.$$

And with more generality,

$$\mathcal{MC}^{\text{new}}(G) = \bigcup_{g \geq 2} \mathcal{MC}_g^{\text{new}}(G).$$

Theorem

Let $G \geq 2$ be an integer. Then $\mathcal{MC}^{new}(G)$ is finite and computable.

M.H. Baker, E. González-Jiménez, J. González, and B. Poonen., *Finiteness results for modular curves of genus at least 2*, Amer. J. Math. **127** (2005), no. 6, 1325–1387.

Notation

Let $G, g \geq 2$ be positive integers, we denote by

$$\mathcal{MC}_g^{\text{new}}(G) = \{[C] \in \mathcal{MC}_g^{\text{new}} \mid C \text{ has gonality } G\}.$$

And with more generality,

$$\mathcal{MC}^{\text{new}}(G) = \bigcup_{g\geq 2} \mathcal{MC}_g^{\text{new}}(G).$$

Theorem

Let $G \geq 2$ be an integer. Then $\mathcal{MC}^{new}(G)$ is finite and computable.

M.H. Baker, E. González-Jiménez, J. González, and B. Poonen., *Finiteness results for modular curves of genus at least 2*, Amer. J. Math. **127** (2005), no. 6, 1325–1387.

Notation

Let $G, g \geq 2$ be positive integers, we denote by

$$\mathcal{M}\mathcal{C}_g^{\text{new}}(G) = \{[C] \in \mathcal{M}\mathcal{C}_g^{\text{new}} \mid C \text{ has gonality } G\}.$$

And with more generality,

$$\mathcal{M}\mathcal{C}^{\text{new}}(G) = \bigcup_{g \geq 2} \mathcal{M}\mathcal{C}_g^{\text{new}}(G).$$

Theorem

Let $G \geq 2$ be an integer. Then $\mathcal{M}\mathcal{C}^{new}(G)$ is finite and computable.

M.H. Baker, E. González-Jiménez, J. González, and B. Poonen., *Finiteness results for modular curves of genus at least 2*, Amer. J. Math. **127** (2005), no. 6, 1325–1387.

Notation

Let $G, g \geq 2$ be positive integers, we denote by

$$\mathcal{MC}_g^{\text{new}}(G) = \{[C] \in \mathcal{MC}_g^{\text{new}} \mid C \text{ has gonality } G\}.$$

And with more generality,

$$\mathcal{MC}^{\text{new}}(G) = \bigcup_{g \geq 2} \mathcal{MC}_g^{\text{new}}(G).$$

Theorem

Let $G \geq 2$ be an integer. Then $\mathcal{MC}^{\text{new}}(G)$ is finite and computable.

M.H. Baker, E. González-Jiménez, J. González, and B. Poonen., *Finiteness results for modular curves of genus at least 2*, Amer. J. Math. **127** (2005), no. 6, 1325–1387.

Conjecture 1

Let $g \geq 2$ be an integer. Then $\#\mathcal{M}\mathcal{C}_g < \infty$

Conjecture 2

Let $G \geq 2$ be an integer. Then $\#\mathcal{M}\mathcal{C}(G) < \infty$

Conjecture 1

Let $g \geq 2$ be an integer. Then $\#\mathcal{M}\mathcal{C}_g < \infty$

Conjecture 2

Let $G \geq 2$ be an integer. Then $\#\mathcal{M}\mathcal{C}(G) < \infty$

Equations for Modular Curves

Hyperelliptic Curves

Definition

Let $C_{/\mathbb{Q}}$ be a curve of genus $g \geq 2$, C is *hyperelliptic* if $\exists x, y \in \mathbb{Q}(C)$ such that $\mathbb{Q}(C) = \mathbb{Q}(x, y)$ and C has an affine model

$$y^2 = F(x),$$

where $F(X) \in \mathbb{Q}[X]$ separable with $\deg F = 2g + 1$ or $2g + 2$.

Hyperelliptic Curves

Definition

Let $C_{/\mathbb{Q}}$ be a curve of genus $g \geq 2$, C is *hyperelliptic* if $\exists\, x, y \in \mathbb{Q}(C)$ such that $\mathbb{Q}(C) = \mathbb{Q}(x, y)$ and C has an affine model

$$y^2 = F(x),$$

where $F(X) \in \mathbb{Q}[X]$ separable with $\deg F = 2g+1$ or $2g+2$.

Hyperelliptic Curves

Definition

Let $C_{/\mathbb{Q}}$ be a curve of genus $g \geq 2$, C is *hyperelliptic* if $\exists\, x, y \in \mathbb{Q}(C)$ such that $\mathbb{Q}(C) = \mathbb{Q}(x, y)$ and C has an affine model

$$y^2 = F(x),$$

where $F(X) \in \mathbb{Q}[X]$ separable with $\deg F = 2g+1$ or $2g+2$.

Hyperelliptic Curves

Definition

Let $C_{/\mathbb{Q}}$ be a curve of genus $g \geq 2$, C is *hyperelliptic* if $\exists\, x, y \in \mathbb{Q}(C)$ such that $\mathbb{Q}(C) = \mathbb{Q}(x, y)$ and C has an affine model

$$y^2 = F(x),$$

where $F(X) \in \mathbb{Q}[X]$ separable with $\deg F = 2g+1$ or $2g+2$.

Hyperelliptic Curves

Definition

Let $C_{/\mathbb{Q}}$ be a curve of genus $g \geq 2$, C is *hyperelliptic* if $\exists\, x, y \in \mathbb{Q}(C)$ such that $\mathbb{Q}(C) = \mathbb{Q}(x, y)$ and C has an affine model

$$y^2 = F(x),$$

where $F(X) \in \mathbb{Q}[X]$ separable with $\deg F = 2g+1$ or $2g+2$.

Hyperelliptic Curves

Definition

Let $C_{/\mathbb{Q}}$ be a curve of genus $g \geq 2$, C is *hyperelliptic* if $\exists\, x, y \in \mathbb{Q}(C)$ such that $\mathbb{Q}(C) = \mathbb{Q}(x, y)$ and C has an affine model

$$y^2 = F(x),$$

where $F(X) \in \mathbb{Q}[X]$ separable with $\deg F = 2g + 1$ or $2g + 2$.

Algorithm

INPUT: $f_1, \ldots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 2$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \ldots h_g\}$ of $H^0(A, \Omega^1)$. Using Gauss elimination check if $\forall\ 1 \leq i \leq g$:

$$\begin{cases} h_i = q^i \qquad + O(q^{g+1}) \\ or \\ h_i = q^{2i-1} + O(q^{2g}) \end{cases}$$

Step 2: Compute $x = \dfrac{h_{g-1}}{h_g}$ and $y = \dfrac{q\,dx/dq}{h_g}$.

Algorithm

INPUT: $f_1, \ldots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 2$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \ldots h_g\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if $\forall\ 1 \leq i \leq g$:

$$\begin{cases} h_i = q^i \qquad + O(q^{g+1}) \\ or \\ h_i = q^{2i-1} + O(q^{2g}) \end{cases}$$

Step 2: Compute $x = \dfrac{h_{g-1}}{h_g}$ and $y = \dfrac{q\,dx/dq}{h_g}$.

Algorithm

INPUT: $f_1, \ldots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 2$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \ldots h_g\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if $\forall\ 1 \leq i \leq g$:

$$\begin{cases} h_i = q^i \qquad + O(q^{g+1}) \\ or \\ h_i = q^{2i-1} + O(q^{2g}) \end{cases}$$

Step 2: Compute $x = \dfrac{h_{g-1}}{h_g}$ and $y = \dfrac{q\, dx/dq}{h_g}$.

Algorithm

INPUT: $f_1, \ldots, f_n \in \text{New}_N$ such that $\dim A = g \geq 2$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \ldots h_g\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if $\forall\ 1 \leq i \leq g$:

$$\begin{cases} h_i = q^i \qquad + O(q^{g+1}) \\ or \\ h_i = q^{2i-1} + O(q^{2g}) \end{cases}$$

Step 2: Compute $x = \dfrac{h_{g-1}}{h_g}$ and $y = \dfrac{q\,dx/dq}{h_g}$.

Algorithm

INPUT: $f_1, \ldots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 2$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \ldots h_g\}$ of $H^0(A, \Omega^1)$. Using Gauss elimination check if $\forall\ 1 \leq i \leq g$:

$$\begin{cases} h_i = q^i \quad + O(q^{g+1}) \\ or \\ h_i = q^{2i-1} + O(q^{2g}) \end{cases}$$

Step 2: Compute $x = \dfrac{h_{g-1}}{h_g}$ and $y = \dfrac{q\,dx/dq}{h_g}$.

Algorithm

INPUT: $f_1, \dots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 2$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \dots h_g\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if $\forall\ 1 \leq i \leq g$:

$$\begin{cases} h_i = q^i \qquad + O(q^{g+1}) \\ or \\ h_i = q^{2i-1} + O(q^{2g}) \end{cases}$$

Step 2: Compute $x = \dfrac{h_{g-1}}{h_g}$ and $y = \dfrac{q\,dx/dq}{h_g}$.

Algorithm

INPUT: $f_1, \ldots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 2$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \ldots h_g\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if $\forall\ 1 \leq i \leq g$:

$$\begin{cases} h_i = q^i \qquad + O(q^{g+1}) \\ or \\ h_i = q^{2i-1} + O(q^{2g}) \end{cases}$$

Step 2: Compute $x = \dfrac{h_{g-1}}{h_g}$ and $y = \dfrac{q\, dx/dq}{h_g}$.

Algorithm

INPUT: $f_1, \ldots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 2$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \ldots h_g\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if $\forall\ 1 \leq i \leq g$:

$$\begin{cases} h_i = q^i \qquad + O(q^{g+1}) \\ or \\ h_i = q^{2i-1} + O(q^{2g}) \end{cases}$$

Step 2: Compute $x = \dfrac{h_{g-1}}{h_g}$ and $y = \dfrac{q\,dx/dq}{h_g}$.

Algorithm

INPUT: $f_1, \ldots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 2$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \ldots h_g\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if $\forall\ 1 \leq i \leq g$:

$$\begin{cases} h_i = q^i \quad\quad + O(q^{g+1}) \\ or \\ h_i = q^{2i-1} + O(q^{2g}) \end{cases}$$

Step 2: Compute $x = \dfrac{h_{g-1}}{h_g}$ and $y = \dfrac{q\,dx/dq}{h_g}$.

Algorithm

INPUT: $f_1, \dots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 2$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \dots h_g\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if $\forall\ 1 \leq i \leq g$:

$$\begin{cases} h_i = q^i \qquad + O(q^{g+1}) \\ or \\ h_i = q^{2i-1} + O(q^{2g}) \end{cases}$$

Step 2: Compute $x = \dfrac{h_{g-1}}{h_g}$ and $y = \dfrac{q\,dx/dq}{h_g}$.

Algorithm

INPUT: $f_1, \ldots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 2$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \ldots h_g\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if $\forall\ 1 \leq i \leq g$:

$$\begin{cases} h_i = q^i \qquad + O(q^{g+1}) \\ or \\ h_i = q^{2i-1} + O(q^{2g}) \end{cases}$$

Step 2: Compute $x = \dfrac{h_{g-1}}{h_g}$ and $y = \dfrac{q\,dx/dq}{h_g}$.

Algorithm (cont.)

Step 3: If $\exists\, F(X) \in \mathbb{Q}[X]$ of degree $2g+1$ or $2g+2$ without multiple roots such that

$$y^2 - F(x) = O(q^{c_N}),\ c_N = 4(g+1)(g_{X_1(N)} - 1) + 1,$$

then $C : y^2 = F(x)$ is a new modular hyperelliptic curve of level N such that

$$J(C) \overset{\mathbb{Q}}{\sim} A.$$

OUTPUT: $C : y^2 = F(x)$ or ERROR.

Algorithm (cont.)

Step 3: If $\exists\, F(X) \in \mathbb{Q}[X]$ of degree $2g+1$ or $2g+2$ without multiple roots such that

$$y^2 - F(x) = O(q^{c_N}),\ c_N = 4(g+1)(g_{X_1(N)} - 1) + 1,$$

then $C : y^2 = F(x)$ is a new modular hyperelliptic curve of level N such that

$$J(C) \overset{\mathbb{Q}}{\sim} A.$$

OUTPUT: $C : y^2 = F(x)$ or ERROR.

Algorithm (cont.)

Step 3: If $\exists\, F(X) \in \mathbb{Q}[X]$ of degree $2g+1$ or $2g+2$ without multiple roots such that

$$y^2 - F(x) = O(q^{c_N}),\ c_N = 4(g+1)(g_{X_1(N)} - 1) + 1,$$

then $C : y^2 = F(x)$ is a new modular hyperelliptic curve of level N such that

$$J(C) \overset{\mathbb{Q}}{\sim} A.$$

OUTPUT: $C : y^2 = F(x)$ or ERROR.

Algorithm (cont.)

Step 3: If $\exists\, F(X) \in \mathbb{Q}[X]$ of degree $2g+1$ or $2g+2$ without multiple roots such that

$$y^2 - F(x) = O(q^{c_N}),\; c_N = 4(g+1)(g_{X_1(N)} - 1) + 1,$$

then $C : y^2 = F(x)$ is a new modular hyperelliptic curve of level N such that

$$J(C) \overset{\mathbb{Q}}{\sim} A.$$

OUTPUT: $C : y^2 = F(x)$ or ERROR.

Algorithm (cont.)

Step 3: If $\exists\, F(X) \in \mathbb{Q}[X]$ of degree $2g+1$ or $2g+2$ without multiple roots such that

$$y^2 - F(x) = O(q^{c_N}),\ c_N = 4(g+1)(g_{X_1(N)} - 1) + 1,$$

then $C : y^2 = F(x)$ is a new modular hyperelliptic curve of level N such that

$$J(C) \stackrel{\mathbb{Q}}{\sim} A.$$

OUTPUT: $C : y^2 = F(x)$ or ERROR.

Algorithm (cont.)

Step 3: If $\exists\, F(X) \in \mathbb{Q}[X]$ of degree $2g+1$ or $2g+2$ without multiple roots such that

$$y^2 - F(x) = O(q^{c_N}),\; c_N = 4(g+1)(g_{X_1(N)} - 1) + 1,$$

then $C : y^2 = F(x)$ is a new modular hyperelliptic curve of level N such that

$$J(C) \overset{\mathbb{Q}}{\sim} A.$$

OUTPUT: $C : y^2 = F(x)$ or ERROR.

Algorithm (cont.)

Step 3: If $\exists\, F(X) \in \mathbb{Q}[X]$ of degree $2g+1$ or $2g+2$ without multiple roots such that

$$y^2 - F(x) = O(q^{c_N}),\; c_N = 4(g+1)(g_{X_1(N)} - 1) + 1,$$

then $C : y^2 = F(x)$ is a new modular hyperelliptic curve of level N such that

$$J(C) \overset{\mathbb{Q}}{\sim} A.$$

OUTPUT: $C : y^2 = F(x)$ or ERROR.

```
Magma V2.14-1   Wed Jul 31 2007 15:15:22 [Seed =3629778794]
   Type ? for help.  Type <Ctrl>-D to quit.

> A:=Af(chi(16,[0,1]));
> A;
[
 Modular symbols space of level 16, weight 2, character $.2, and
  dimension 1 over Cyclotomic Field of order 4 and degree 2
]
> NewModularHyperellipticCurve(A);
x^6 + 2*x^5 - x^4 - x^2 - 2*x + 1
```

```
Magma V2.14-1   Wed Jul 31 2007 15:15:22 [Seed =3629778794]
   Type ? for help.  Type <Ctrl>-D to quit.

> A:=Af(chi(16,[0,1]));
> A;
[
 Modular symbols space of level 16, weight 2, character $.2, and
  dimension 1 over Cyclotomic Field of order 4 and degree 2
]
> NewModularHyperellipticCurve(A);
x^6 + 2*x^5 - x^4 - x^2 - 2*x + 1
```

```
Magma V2.14-1   Wed Jul 31 2007 15:15:22 [Seed =3629778794]
   Type ? for help.  Type <Ctrl>-D to quit.

> A:=Af(chi(16,[0,1]));
> A;
[
 Modular symbols space of level 16, weight 2, character $.2, and
  dimension 1 over Cyclotomic Field of order 4 and degree 2
]
> NewModularHyperellipticCurve(A);
x^6 + 2*x^5 - x^4 - x^2 - 2*x + 1
```

```
Magma V2.14-1   Wed Jul 31 2007 15:15:22 [Seed =3629778794]
   Type ? for help.  Type <Ctrl>-D to quit.
> A:=Af(chi(16,[0,1]));
> A;
[
 Modular symbols space of level 16, weight 2, character $.2, and
  dimension 1 over Cyclotomic Field of order 4 and degree 2
]
> NewModularHyperellipticCurve(A);
x^6 + 2*x^5 - x^4 - x^2 - 2*x + 1
```

```
Magma V2.14-1   Wed Jul 31 2007 15:15:22 [Seed =3629778794]
   Type ? for help.  Type <Ctrl>-D to quit.
> A:=Af(chi(16,[0,1]));
> A;
[
 Modular symbols space of level 16, weight 2, character $.2, and
  dimension 1 over Cyclotomic Field of order 4 and degree 2
]
> NewModularHyperellipticCurve(A);
x^6 + 2*x^5 - x^4 - x^2 - 2*x + 1
```

```
> B:=Af(23);B;
[
    Modular symbols space for Gamma_0(23) of weight 2 and dimension 2 ove
]
> f:=qEigenform(B[1],6);f;E_Field(B[1]);
q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + O(q^6)
Number Field with defining polynomial x^2 + x - 1 over the
Rational Field
> ff:=qIntegralBasis(B[1],20);ff;
[
 q - q^3 - q^4 - 2*q^6 + 2*q^7 - q^8 + 2*q^9 + 2*q^10 - 4*q^11 +
3*q^12 + 3*q^13 + 2*q^14 - 4*q^15 + 2*q^17 - 2*q^19 + O(q^20),
  q^2 - 2*q^3 - q^4 + 2*q^5 + q^6 + 2*q^7 - 2*q^8 - 2*q^10 - 2*q^11
+ q^12 + 2*q^15 + 3*q^16 - 2*q^17 + 2*q^18 + O(q^20)
]
> NewModularHyperellipticCurve(ff);
x^6 + 4*x^5 - 18*x^4 - 142*x^3 - 351*x^2 - 394*x - 175
```

```
> B:=Af(23);B;
[
    Modular symbols space for Gamma_0(23) of weight 2 and dimension 2
]
> f:=qEigenform(B[1],6);f;E_Field(B[1]);
q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + O(q^6)
Number Field with defining polynomial x^2 + x - 1 over the
Rational Field
> ff:=qIntegralBasis(B[1],20);ff;
[
 q - q^3 - q^4 - 2*q^6 + 2*q^7 - q^8 + 2*q^9 + 2*q^10 - 4*q^11 +
3*q^12 + 3*q^13 + 2*q^14 - 4*q^15 + 2*q^17 - 2*q^19 + O(q^20),
  q^2 - 2*q^3 - q^4 + 2*q^5 + q^6 + 2*q^7 - 2*q^8 - 2*q^10 - 2*q^11
+ q^12 + 2*q^15 + 3*q^16 - 2*q^17 + 2*q^18 + O(q^20)
]
> NewModularHyperellipticCurve(ff);
x^6 + 4*x^5 - 18*x^4 - 142*x^3 - 351*x^2 - 394*x - 175
```

```
> B:=Af(23);B;
[
    Modular symbols space for Gamma_0(23) of weight 2 and dimension 2
]
> f:=qEigenform(B[1],6);f;E_Field(B[1]);
q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + O(q^6)
Number Field with defining polynomial x^2 + x - 1 over the
Rational Field
> ff:=qIntegralBasis(B[1],20);ff;
[
 q - q^3 - q^4 - 2*q^6 + 2*q^7 - q^8 + 2*q^9 + 2*q^10 - 4*q^11 +
3*q^12 + 3*q^13 + 2*q^14 - 4*q^15 + 2*q^17 - 2*q^19 + O(q^20),
  q^2 - 2*q^3 - q^4 + 2*q^5 + q^6 + 2*q^7 - 2*q^8 - 2*q^10 - 2*q^11
+ q^12 + 2*q^15 + 3*q^16 - 2*q^17 + 2*q^18 + O(q^20)
]
> NewModularHyperellipticCurve(ff);
x^6 + 4*x^5 - 18*x^4 - 142*x^3 - 351*x^2 - 394*x - 175
```

```
> B:=Af(23);B;
[
    Modular symbols space for Gamma_0(23) of weight 2 and dimension 2
]
> f:=qEigenform(B[1],6);f;E_Field(B[1]);
q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + O(q^6)
Number Field with defining polynomial x^2 + x - 1 over the
Rational Field
> ff:=qIntegralBasis(B[1],20);ff;
[
 q - q^3 - q^4 - 2*q^6 + 2*q^7 - q^8 + 2*q^9 + 2*q^10 - 4*q^11 +
3*q^12 + 3*q^13 + 2*q^14 - 4*q^15 + 2*q^17 - 2*q^19 + O(q^20),
  q^2 - 2*q^3 - q^4 + 2*q^5 + q^6 + 2*q^7 - 2*q^8 - 2*q^10 - 2*q^11
+ q^12 + 2*q^15 + 3*q^16 - 2*q^17 + 2*q^18 + O(q^20)
]
> NewModularHyperellipticCurve(ff);
x^6 + 4*x^5 - 18*x^4 - 142*x^3 - 351*x^2 - 394*x - 175
```

```
> B:=Af(23);B;
[
    Modular symbols space for Gamma_0(23) of weight 2 and dimension 2
]
> f:=qEigenform(B[1],6);f;E_Field(B[1]);
q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + O(q^6)
Number Field with defining polynomial x^2 + x - 1 over the
Rational Field
> ff:=qIntegralBasis(B[1],20);ff;
[
 q - q^3 - q^4 - 2*q^6 + 2*q^7 - q^8 + 2*q^9 + 2*q^10 - 4*q^11 +
3*q^12 + 3*q^13 + 2*q^14 - 4*q^15 + 2*q^17 - 2*q^19 + O(q^20),
  q^2 - 2*q^3 - q^4 + 2*q^5 + q^6 + 2*q^7 - 2*q^8 - 2*q^10 - 2*q^11
+ q^12 + 2*q^15 + 3*q^16 - 2*q^17 + 2*q^18 + O(q^20)
]
> NewModularHyperellipticCurve(ff);
x^6 + 4*x^5 - 18*x^4 - 142*x^3 - 351*x^2 - 394*x - 175
```

```
> B:=Af(23);B;
[
    Modular symbols space for Gamma_0(23) of weight 2 and dimension 2
]
> f:=qEigenform(B[1],6);f;E_Field(B[1]);
q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + O(q^6)
Number Field with defining polynomial x^2 + x - 1 over the
Rational Field
> ff:=qIntegralBasis(B[1],20);ff;
[
 q - q^3 - q^4 - 2*q^6 + 2*q^7 - q^8 + 2*q^9 + 2*q^10 - 4*q^11 +
3*q^12 + 3*q^13 + 2*q^14 - 4*q^15 + 2*q^17 - 2*q^19 + O(q^20),
  q^2 - 2*q^3 - q^4 + 2*q^5 + q^6 + 2*q^7 - 2*q^8 - 2*q^10 - 2*q^11
+ q^12 + 2*q^15 + 3*q^16 - 2*q^17 + 2*q^18 + O(q^20)
]
> NewModularHyperellipticCurve(ff);
x^6 + 4*x^5 - 18*x^4 - 142*x^3 - 351*x^2 - 394*x - 175
```

```
> B:=Af(23);B;
[
    Modular symbols space for Gamma_0(23) of weight 2 and dimension 2
]
> f:=qEigenform(B[1],6);f;E_Field(B[1]);
q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + O(q^6)
Number Field with defining polynomial x^2 + x - 1 over the
Rational Field
> ff:=qIntegralBasis(B[1],20);ff;
[
 q - q^3 - q^4 - 2*q^6 + 2*q^7 - q^8 + 2*q^9 + 2*q^10 - 4*q^11 +
3*q^12 + 3*q^13 + 2*q^14 - 4*q^15 + 2*q^17 - 2*q^19 + O(q^20),
  q^2 - 2*q^3 - q^4 + 2*q^5 + q^6 + 2*q^7 - 2*q^8 - 2*q^10 - 2*q^11
+ q^12 + 2*q^15 + 3*q^16 - 2*q^17 + 2*q^18 + O(q^20)
]
> NewModularHyperellipticCurve(ff);
x^6 + 4*x^5 - 18*x^4 - 142*x^3 - 351*x^2 - 394*x - 175
```

```
> NewModularHyperellipticCurves;

Intrinsic ’NewModularHyperellipticCurves’

Signatures:

    (<RngIntElt> N) -> SeqEnum
    [
        prec,
        verbose,
        gamma,
        genus,
        check
    ]

        All the New Modular Hyperelliptic Curve of level N.
```

```
> NewModularHyperellipticCurves;

Intrinsic 'NewModularHyperellipticCurves'

Signatures:

    (<RngIntElt> N) -> SeqEnum
    [
        prec,
        verbose,
        gamma,
        genus,
        check
    ]

        All the New Modular Hyperelliptic Curve of level N.
```

```
> NewModularHyperellipticCurves(16);
[
    x^6 + 2*x^5 - x^4 - x^2 - 2*x + 1
]
```

```
> NewModularHyperellipticCurves(16);
[
    x^6 + 2*x^5 - x^4 - x^2 - 2*x + 1
]
```

```
> NewModularHyperellipticCurves(376:gamma:=0,verbose:=1);

     Candidates:=14

     Maximum g:=10

1 2 3 4 5 6 7 8 9 10 11 12 13 14

[
 x^9+2*x^8-8*x^7-16*x^6+9*x^5+3*x^4-36*x^3+x^2+12*x-20,
 x^9-2*x^8-4*x^7+8*x^6-7*x^5-5*x^4+16*x^3-19*x^2+12*x-4,
 x^5+4*x^4+3*x^3-2*x^2+2*x+5,
 x^5-x^3+2*x^2-2*x+1
]
```

```
> NewModularHyperellipticCurves(376:gamma:=0,verbose:=1);

     Candidates:=14

     Maximum g:=10

1  2  3  4  5  6  7  8  9  10  11  12  13  14

[
 x^9+2*x^8-8*x^7-16*x^6+9*x^5+3*x^4-36*x^3+x^2+12*x-20,
 x^9-2*x^8-4*x^7+8*x^6-7*x^5-5*x^4+16*x^3-19*x^2+12*x-4,
 x^5+4*x^4+3*x^3-2*x^2+2*x+5,
 x^5-x^3+2*x^2-2*x+1
]
```

$$C \quad : \quad y^2 \; = \; F(x)$$

$$
\begin{array}{lcl}
C^A_{21A_{\{0,2\}}} & : & y^2 = (x^2 - x + 1)(x^6 + x^5 - 6x^4 - 3x^3 + 14x^2 - 7x + 1) \\
X_0(35) & : & y^2 = (x^2 + 3x + 1)(x^6 + x^5 - 10x^4 - 39x^3 - 62x^2 - 51x - 19) \\
C^A_{36A_{\{0,2\}}} & : & y^2 = (x + 1)(x + 2)(x^2 + 3x + 3)(x^3 - 9x - 9) \\
X_0(39) & : & y^2 = (x^4 - 3x^3 - 4x^2 - 2x - 1)(x^4 + 5x^3 + 8x^2 + 6x + 3) \\
\vdots & & \vdots
\end{array}
$$

$\boxed{\text{Total: 87 curves}}$

$$
\begin{array}{lcl}
\vdots & & \vdots \\
C_{1136I} & : & y^2 = x^7 - 4x^6 + 5x^5 - x^4 - 3x^3 + 2x^2 - 1 \\
C_{1136J} & : & y^2 = x^7 - 7x^5 + 11x^4 + 5x^3 - 18x^2 + 4x + 11 \\
C_{1664Y} & : & y^2 = (x^2 + 2x + 2)(x^3 - x + 2)(x^6 + 2x^4 + x^2 + 4) \\
C_{1664BB} & : & y^2 = (x^2 - 2x + 2)(x^3 - x - 2)(x^6 + 2x^4 + x^2 + 4)
\end{array}
$$

C	:	$y^2 = F(x)$
$C^A_{21A_{\{0,2\}}}$	:	$y^2 = (x^2 - x + 1)(x^6 + x^5 - 6x^4 - 3x^3 + 14x^2 - 7x + 1)$
$X_0(35)$	:	$y^2 = (x^2 + 3x + 1)(x^6 + x^5 - 10x^4 - 39x^3 - 62x^2 - 51x - 19)$
$C^A_{36A_{\{0,2\}}}$	:	$y^2 = (x + 1)(x + 2)(x^2 + 3x + 3)(x^3 - 9x - 9)$
$X_0(39)$	:	$y^2 = (x^4 - 3x^3 - 4x^2 - 2x - 1)(x^4 + 5x^3 + 8x^2 + 6x + 3)$
$\vdots$		$\vdots$
		Total: 87 curves
$\vdots$		$\vdots$
C_{1136I}	:	$y^2 = x^7 - 4x^6 + 5x^5 - x^4 - 3x^3 + 2x^2 - 1$
C_{1136J}	:	$y^2 = x^7 - 7x^5 + 11x^4 + 5x^3 - 18x^2 + 4x + 11$
C_{1664Y}	:	$y^2 = (x^2 + 2x + 2)(x^3 - x + 2)(x^6 + 2x^4 + x^2 + 4)$
C_{1664BB}	:	$y^2 = (x^2 - 2x + 2)(x^3 - x - 2)(x^6 + 2x^4 + x^2 + 4)$

C	:	$y^2 = F(x)$
$C^A_{21A_{\{0,2\}}}$	:	$y^2 = (x^2 - x + 1)(x^6 + x^5 - 6x^4 - 3x^3 + 14x^2 - 7x + 1)$
$X_0(35)$	:	$y^2 = (x^2 + 3x + 1)(x^6 + x^5 - 10x^4 - 39x^3 - 62x^2 - 51x - 19)$
$C^A_{36A_{\{0,2\}}}$	:	$y^2 = (x+1)(x+2)(x^2+3x+3)(x^3-9x-9)$
$X_0(39)$	:	$y^2 = (x^4 - 3x^3 - 4x^2 - 2x - 1)(x^4 + 5x^3 + 8x^2 + 6x + 3)$
$\vdots$		$\vdots$
		Total: 87 curves
$\vdots$		$\vdots$
C_{1136I}	:	$y^2 = x^7 - 4x^6 + 5x^5 - x^4 - 3x^3 + 2x^2 - 1$
C_{1136J}	:	$y^2 = x^7 - 7x^5 + 11x^4 + 5x^3 - 18x^2 + 4x + 11$
C_{1664Y}	:	$y^2 = (x^2 + 2x + 2)(x^3 - x + 2)(x^6 + 2x^4 + x^2 + 4)$
C_{1664BB}	:	$y^2 = (x^2 - 2x + 2)(x^3 - x - 2)(x^6 + 2x^4 + x^2 + 4)$

C	:	$y^2 = F(x)$
$C^A_{21A_{\{0,2\}}}$	:	$y^2 = (x^2 - x + 1)(x^6 + x^5 - 6x^4 - 3x^3 + 14x^2 - 7x + 1)$
$X_0(35)$	:	$y^2 = (x^2 + 3x + 1)(x^6 + x^5 - 10x^4 - 39x^3 - 62x^2 - 51x - 19)$
$C^A_{36A_{\{0,2\}}}$	:	$y^2 = (x + 1)(x + 2)(x^2 + 3x + 3)(x^3 - 9x - 9)$
$X_0(39)$	:	$y^2 = (x^4 - 3x^3 - 4x^2 - 2x - 1)(x^4 + 5x^3 + 8x^2 + 6x + 3)$
$\vdots$		$\vdots$
		Total: 87 curves
$\vdots$		$\vdots$
C_{1136I}	:	$y^2 = x^7 - 4x^6 + 5x^5 - x^4 - 3x^3 + 2x^2 - 1$
C_{1136J}	:	$y^2 = x^7 - 7x^5 + 11x^4 + 5x^3 - 18x^2 + 4x + 11$
C_{1664Y}	:	$y^2 = (x^2 + 2x + 2)(x^3 - x + 2)(x^6 + 2x^4 + x^2 + 4)$
C_{1664BB}	:	$y^2 = (x^2 - 2x + 2)(x^3 - x - 2)(x^6 + 2x^4 + x^2 + 4)$

C	:	$y^2 = F(x)$
$C^A_{21A_{\{0,2\}}}$	:	$y^2 = (x^2 - x + 1)(x^6 + x^5 - 6x^4 - 3x^3 + 14x^2 - 7x + 1)$
$X_0(35)$	:	$y^2 = (x^2 + 3x + 1)(x^6 + x^5 - 10x^4 - 39x^3 - 62x^2 - 51x - 19)$
$C^A_{36A_{\{0,2\}}}$	:	$y^2 = (x + 1)(x + 2)(x^2 + 3x + 3)(x^3 - 9x - 9)$
$X_0(39)$	:	$y^2 = (x^4 - 3x^3 - 4x^2 - 2x - 1)(x^4 + 5x^3 + 8x^2 + 6x + 3)$
$\vdots$		$\vdots$
		Total: 87 curves
$\vdots$		$\vdots$
C_{1136I}	:	$y^2 = x^7 - 4x^6 + 5x^5 - x^4 - 3x^3 + 2x^2 - 1$
C_{1136J}	:	$y^2 = x^7 - 7x^5 + 11x^4 + 5x^3 - 18x^2 + 4x + 11$
C_{1664Y}	:	$y^2 = (x^2 + 2x + 2)(x^3 - x + 2)(x^6 + 2x^4 + x^2 + 4)$
C_{1664BB}	:	$y^2 = (x^2 - 2x + 2)(x^3 - x - 2)(x^6 + 2x^4 + x^2 + 4)$

$$
\begin{array}{lcl}
C & : & y^2 = F(x) \\
C^A_{21A_{\{0,2\}}} & : & y^2=(x^2-x+1)(x^6+x^5-6x^4-3x^3+14x^2-7x+1) \\
X_0(35) & : & y^2=(x^2+3x+1)(x^6+x^5-10x^4-39x^3-62x^2-51x-19) \\
C^A_{36A_{\{0,2\}}} & : & y^2=(x+1)(x+2)(x^2+3x+3)(x^3-9x-9) \\
X_0(39) & : & y^2=(x^4-3x^3-4x^2-2x-1)(x^4+5x^3+8x^2+6x+3) \\
\vdots & & \vdots
\end{array}
$$

$$\boxed{\text{Total: 87 curves}}$$

$$
\begin{array}{lcl}
\vdots & & \vdots \\
C_{1136I} & : & y^2=x^7-4x^6+5x^5-x^4-3x^3+2x^2-1 \\
C_{1136J} & : & y^2=x^7-7x^5+11x^4+5x^3-18x^2+4x+11 \\
C_{1664Y} & : & y^2=(x^2+2x+2)(x^3-x+2)(x^6+2x^4+x^2+4) \\
C_{1664BB} & : & y^2=(x^2-2x+2)(x^3-x-2)(x^6+2x^4+x^2+4)
\end{array}
$$

C	:	$y^2 = F(x)$
$C^A_{21A_{\{0,2\}}}$	:	$y^2 = (x^2 - x + 1)(x^6 + x^5 - 6x^4 - 3x^3 + 14x^2 - 7x + 1)$
$X_0(35)$	:	$y^2 = (x^2 + 3x + 1)(x^6 + x^5 - 10x^4 - 39x^3 - 62x^2 - 51x - 19)$
$C^A_{36A_{\{0,2\}}}$	:	$y^2 = (x+1)(x+2)(x^2+3x+3)(x^3-9x-9)$
$X_0(39)$	:	$y^2 = (x^4 - 3x^3 - 4x^2 - 2x - 1)(x^4 + 5x^3 + 8x^2 + 6x + 3)$
$\vdots$		$\vdots$
		$\boxed{\text{Total: 87 curves}}$
$\vdots$		$\vdots$
C_{1136I}	:	$y^2 = x^7 - 4x^6 + 5x^5 - x^4 - 3x^3 + 2x^2 - 1$
C_{1136J}	:	$y^2 = x^7 - 7x^5 + 11x^4 + 5x^3 - 18x^2 + 4x + 11$
C_{1664Y}	:	$y^2 = (x^2 + 2x + 2)(x^3 - x + 2)(x^6 + 2x^4 + x^2 + 4)$
C_{1664BB}	:	$y^2 = (x^2 - 2x + 2)(x^3 - x - 2)(x^6 + 2x^4 + x^2 + 4)$

C	:	$y^2 = F(x)$
$C^A_{21A_{\{0,2\}}}$	:	$y^2=(x^2-x+1)(x^6+x^5-6x^4-3x^3+14x^2-7x+1)$
$X_0(35)$	:	$y^2=(x^2+3x+1)(x^6+x^5-10x^4-39x^3-62x^2-51x-19)$
$C^A_{36A_{\{0,2\}}}$	:	$y^2=(x+1)(x+2)(x^2+3x+3)(x^3-9x-9)$
$X_0(39)$	:	$y^2=(x^4-3x^3-4x^2-2x-1)(x^4+5x^3+8x^2+6x+3)$
$\vdots$		$\vdots$
		$\boxed{\text{Total: 87 curves}}$
$\vdots$		$\vdots$
C_{1136I}	:	$y^2=x^7-4x^6+5x^5-x^4-3x^3+2x^2-1$
C_{1136J}	:	$y^2=x^7-7x^5+11x^4+5x^3-18x^2+4x+11$
C_{1664Y}	:	$y^2=(x^2+2x+2)(x^3-x+2)(x^6+2x^4+x^2+4)$
C_{1664BB}	:	$y^2=(x^2-2x+2)(x^3-x-2)(x^6+2x^4+x^2+4)$

$$
\begin{array}{lcl}
C & : & y^2 = F(x) \\
C^A_{21A_{\{0,2\}}} & : & y^2 = (x^2 - x + 1)(x^6 + x^5 - 6x^4 - 3x^3 + 14x^2 - 7x + 1) \\
X_0(35) & : & y^2 = (x^2 + 3x + 1)(x^6 + x^5 - 10x^4 - 39x^3 - 62x^2 - 51x - 19) \\
C^A_{36A_{\{0,2\}}} & : & y^2 = (x+1)(x+2)(x^2 + 3x + 3)(x^3 - 9x - 9) \\
X_0(39) & : & y^2 = (x^4 - 3x^3 - 4x^2 - 2x - 1)(x^4 + 5x^3 + 8x^2 + 6x + 3) \\
\vdots & & \vdots
\end{array}
$$

$$\boxed{\text{Total: 87 curves}}$$

$$
\begin{array}{lcl}
\vdots & & \vdots \\
C_{1136I} & : & y^2 = x^7 - 4x^6 + 5x^5 - x^4 - 3x^3 + 2x^2 - 1 \\
C_{1136J} & : & y^2 = x^7 - 7x^5 + 11x^4 + 5x^3 - 18x^2 + 4x + 11 \\
C_{1664Y} & : & y^2 = (x^2 + 2x + 2)(x^3 - x + 2)(x^6 + 2x^4 + x^2 + 4) \\
C_{1664BB} & : & y^2 = (x^2 - 2x + 2)(x^3 - x - 2)(x^6 + 2x^4 + x^2 + 4)
\end{array}
$$

$$C \quad : \quad y^2 = F(x)$$

$$
\begin{array}{lcl}
C^A_{21A_{\{0,2\}}} & : & y^2 = (x^2 - x + 1)(x^6 + x^5 - 6x^4 - 3x^3 + 14x^2 - 7x + 1) \\
X_0(35) & : & y^2 = (x^2 + 3x + 1)(x^6 + x^5 - 10x^4 - 39x^3 - 62x^2 - 51x - 19) \\
C^A_{36A_{\{0,2\}}} & : & y^2 = (x+1)(x+2)(x^2 + 3x + 3)(x^3 - 9x - 9) \\
X_0(39) & : & y^2 = (x^4 - 3x^3 - 4x^2 - 2x - 1)(x^4 + 5x^3 + 8x^2 + 6x + 3) \\
\vdots & & \vdots
\end{array}
$$

$$\boxed{\text{Total: 87 curves}}$$

$$
\begin{array}{lcl}
\vdots & & \vdots \\
C_{1136I} & : & y^2 = x^7 - 4x^6 + 5x^5 - x^4 - 3x^3 + 2x^2 - 1 \\
C_{1136J} & : & y^2 = x^7 - 7x^5 + 11x^4 + 5x^3 - 18x^2 + 4x + 11 \\
C_{1664Y} & : & y^2 = (x^2 + 2x + 2)(x^3 - x + 2)(x^6 + 2x^4 + x^2 + 4) \\
C_{1664BB} & : & y^2 = (x^2 - 2x + 2)(x^3 - x - 2)(x^6 + 2x^4 + x^2 + 4)
\end{array}
$$

Conjecture

$\#\mathcal{MC}^{\text{new}}(2) = 300$

Non-Hyperelliptic Curves

Let $C_{/\mathbb{Q}}$ be a non-hyperelliptic curve of genus $g \geq 3$, and

$$H^0(C, \Omega^1_C) = \langle \omega_1, \ldots, \omega_g \rangle_{\mathbb{C}} \,.$$

Then there exists the *canonical embedding* defined by:

$$i : C \hookrightarrow \mathbb{P}^{g-1} \;:\; z \mapsto [\omega_1(z) : \cdots : \omega_g(z)]$$

where $i(C)$ is a nonsingular projective curve.

Petri's Theorem:

$$i(C) = \bigcap_{d=2}^{4} \left\{ \mathcal{HS}_d \;\middle|\; i(C) \subset \mathcal{HS}_d \,, \begin{array}{ll} codim(\mathcal{HS}_d) & = 1 \\ deg(\mathcal{HS}_d) & = d \end{array} \right\}$$

Non-Hyperelliptic Curves

Let $C_{/\mathbb{Q}}$ be a non-hyperelliptic curve of genus $g \geq 3$, and

$$H^0(C, \Omega^1_C) = \langle \omega_1, \ldots, \omega_g \rangle_{\mathbb{C}} \,.$$

Then there exists the *canonical embedding* defined by:

$$i : C \hookrightarrow \mathbb{P}^{g-1} \; : \; z \mapsto [\omega_1(z) : \cdots : \omega_g(z)]$$

where $i(C)$ is a nonsingular projective curve.

Petri's Theorem:

$$i(C) = \bigcap_{d=2}^{4} \left\{ \mathcal{HS}_d \,\middle|\, i(C) \subset \mathcal{HS}_d \,, \begin{array}{ll} codim(\mathcal{HS}_d) & = 1 \\ deg(\mathcal{HS}_d) & = d \end{array} \right\}$$

Non-Hyperelliptic Curves

Let $C_{/\mathbb{Q}}$ be a non-hyperelliptic curve of genus $g \geq 3$, and

$$H^0(C, \Omega^1_C) = \langle \omega_1, \dots, \omega_g \rangle_{\mathbb{C}} \,.$$

Then there exists the *canonical embedding* defined by:

$$i : C \hookrightarrow \mathbb{P}^{g-1} \; : \; z \mapsto [\omega_1(z) : \cdots : \omega_g(z)]$$

where $i(C)$ is a nonsingular projective curve.

Petri's Theorem:

$$i(C) = \bigcap_{d=2}^{4} \left\{ \mathcal{HS}_d \,\middle|\, i(C) \subset \mathcal{HS}_d \,, \begin{array}{ll} codim(\mathcal{HS}_d) & = 1 \\ deg(\mathcal{HS}_d) & = d \end{array} \right\}$$

Non-Hyperelliptic Curves

Let $C_{/\mathbb{Q}}$ be a non-hyperelliptic curve of genus $g \geq 3$, and

$$H^0(C, \Omega_C^1) = \langle \omega_1, \ldots, \omega_g \rangle_{\mathbb{C}}.$$

Then there exists the *canonical embedding* defined by:

$$i : C \hookrightarrow \mathbb{P}^{g-1} \;:\; z \mapsto [\omega_1(z) : \cdots : \omega_g(z)]$$

where $i(C)$ is a nonsingular projective curve.

Petri's Theorem:

$$i(C) = \bigcap_{d=2}^{4} \left\{ \mathcal{HS}_d \;\middle|\; i(C) \subset \mathcal{HS}_d\,,\; \begin{array}{ll} codim(\mathcal{HS}_d) & = 1 \\ deg(\mathcal{HS}_d) & = d \end{array} \right\}$$

Non-Hyperelliptic Curves

Let $C_{/\mathbb{Q}}$ be a non-hyperelliptic curve of genus $g \geq 3$, and

$$H^0(C, \Omega^1_C) = \langle \omega_1, \ldots, \omega_g \rangle_{\mathbb{C}}.$$

Then there exists the *canonical embedding* defined by:

$$i : C \hookrightarrow \mathbb{P}^{g-1} \;:\; z \mapsto [\omega_1(z) : \cdots : \omega_g(z)]$$

where $i(C)$ is a nonsingular projective curve.

Petri's Theorem:

$$i(C) = \bigcap_{d=2}^{4} \left\{ \mathcal{HS}_d \;\middle|\; i(C) \subset \mathcal{HS}_d\,, \begin{array}{ll} codim(\mathcal{HS}_d) & = 1 \\ deg(\mathcal{HS}_d) & = d \end{array} \right\}$$

Example

Algorithm $g = 3$ (joint work with Roger Oyono)

INPUT: $f_1, \dots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 3$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \dots h_3\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if

$$\begin{cases} h_1 = & q + \quad O(q^2) \\ h_2 = & \quad\quad q^2 + \quad O(q^3) \\ h_3 = & \quad\quad\quad\quad O(q^3) \end{cases}$$

Step 2: Set coordinates for the canonical embeding

$$\begin{cases} x = h_1 \\ y = h_2 \\ z = h_3 \end{cases}$$

Example

Algorithm $g = 3$ (joint work with Roger Oyono)

INPUT: $f_1, \ldots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 3$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \ldots h_3\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if

$$\begin{cases} h_1 = & q + \quad O(q^2) \\ h_2 = & \qquad q^2 + \quad O(q^3) \\ h_3 = & \qquad\qquad O(q^3) \end{cases}$$

Step 2: Set coordinates for the canonical embeding

$$\begin{cases} x = h_1 \\ y = h_2 \\ z = h_3 \end{cases}$$

Example

Algorithm $g = 3$ (joint work with Roger Oyono)

INPUT: $f_1, \ldots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 3$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \ldots h_3\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if

$$\begin{cases} h_1 = & q+ & O(q^2) & \\ h_2 = & & q^2+ & O(q^3) \\ h_3 = & & & O(q^3) \end{cases}$$

Step 2: Set coordinates for the canonical embeding

$$\begin{cases} x = h_1 \\ y = h_2 \\ z = h_3 \end{cases}$$

Example

Algorithm $g = 3$ (joint work with Roger Oyono)

INPUT: $f_1, \ldots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 3$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \ldots h_3\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if

$$\begin{cases} h_1 = & q+ & O(q^2) & \\ h_2 = & & q^2+ & O(q^3) \\ h_3 = & & & O(q^3) \end{cases}$$

Step 2: Set coordinates for the canonical embeding

$$\begin{cases} x = h_1 \\ y = h_2 \\ z = h_3 \end{cases}$$

Example

Algorithm $g = 3$ (joint work with Roger Oyono)

INPUT: $f_1, \dots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 3$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \dots h_3\}$ of $H^0(A, \Omega^1)$. Using Gauss elimination check if

$$\begin{cases} h_1 = & q + \quad O(q^2) \\ h_2 = & \qquad q^2 + \quad O(q^3) \\ h_3 = & \qquad\qquad O(q^3) \end{cases}$$

Step 2: Set coordinates for the canonical embeding

$$\begin{cases} x = h_1 \\ y = h_2 \\ z = h_3 \end{cases}$$

Example

Algorithm $g = 3$ (joint work with Roger Oyono)

INPUT: $f_1, \ldots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 3$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \ldots h_3\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if

$$\begin{cases} h_1 = & q + & O(q^2) & \\ h_2 = & & q^2 + & O(q^3) \\ h_3 = & & & O(q^3) \end{cases}$$

Step 2: Set coordinates for the canonical embeding

$$\begin{cases} x = h_1 \\ y = h_2 \\ z = h_3 \end{cases}$$

Example

Algorithm $g = 3$ (joint work with Roger Oyono)

INPUT: $f_1, \dots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 3$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \dots h_3\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if

$$\begin{cases} h_1 = & q + & O(q^2) \\ h_2 = & & q^2 + & O(q^3) \\ h_3 = & & & O(q^3) \end{cases}$$

Step 2: Set coordinates for the canonical embeding

$$\begin{cases} x = h_1 \\ y = h_2 \\ z = h_3 \end{cases}$$

Example

Algorithm $g = 3$ (joint work with Roger Oyono)

INPUT: $f_1, \dots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 3$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \dots h_3\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if

$$\begin{cases} h_1 = & q + & O(q^2) & \\ h_2 = & & q^2 + & O(q^3) \\ h_3 = & & & O(q^3) \end{cases}$$

Step 2: Set coordinates for the canonical embeding

$$\begin{cases} x = h_1 \\ y = h_2 \\ z = h_3 \end{cases}$$

Example

Algorithm $g = 3$ (joint work with Roger Oyono)

INPUT: $f_1, \ldots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 3$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \ldots h_3\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if

$$\begin{cases} h_1 = & q+ & O(q^2) & \\ h_2 = & & q^2+ & O(q^3) \\ h_3 = & & & O(q^3) \end{cases}$$

Step 2: Set coordinates for the canonical embeding

$$\begin{cases} x = h_1 \\ y = h_2 \\ z = h_3 \end{cases}$$

Example

Algorithm $g = 3$ (joint work with Roger Oyono)

INPUT: $f_1, \dots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 3$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \dots h_3\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if

$$\begin{cases} h_1 = & q + \quad O(q^2) \\ h_2 = & \qquad q^2 + \quad O(q^3) \\ h_3 = & \qquad\qquad O(q^3) \end{cases}$$

Step 2: Set coordinates for the canonical embeding

$$\begin{cases} x = h_1 \\ y = h_2 \\ z = h_3 \end{cases}$$

Example

Algorithm $g = 3$ (joint work with Roger Oyono)

INPUT: $f_1, \ldots, f_n \in \mathrm{New}_N$ such that $\dim A = g \geq 3$, $A = A_{f_1} \times \cdots \times A_{f_n}$.

Step 1: Compute a rational basis $\{h_1, \ldots h_3\}$ of $H^0(A, \Omega^1)$.Using Gauss elimination check if

$$\begin{cases} h_1 = & q + \quad O(q^2) \\ h_2 = & \qquad q^2 + \quad O(q^3) \\ h_3 = & \qquad\qquad\quad O(q^3) \end{cases}$$

Step 2: Set coordinates for the canonical embeding

$$\begin{cases} x = h_1 \\ y = h_2 \\ z = h_3 \end{cases}$$

Algorithm (cont.)

Step 3: Compute if there exists

$$F(X,Y,Z)=\sum_{i+j+k=4} a_{ijk}X^iY^jZ^k \in \mathbb{Q}[X,Y,Z]$$

such that

$$F(x,y,z)=O(q^{c_N}),\ c_N=\frac{4}{3}[SL_2(\mathbb{Z}):\Gamma_1(N)],$$

Step 4: If $C: F(X,Y,Z)=0$ is smooth and of genus 3 then C is a non-hyperelliptic modular curve of genus 3, level N such that

$$J(C)\overset{\mathbb{Q}}{\sim} A.$$

OUTPUT: $C: F(X,Y,Z)=0$ or ERROR.

Algorithm (cont.)

Step 3: Compute if there exists

$$F(X,Y,Z)=\sum_{i+j+k=4} a_{ijk}X^iY^jZ^k \in \mathbb{Q}[X,Y,Z]$$

such that

$$F(x,y,z)=O(q^{c_N}),\ c_N=\frac{4}{3}[SL_2(\mathbb{Z}):\Gamma_1(N)],$$

Step 4: If $C: F(X,Y,Z)=0$ is smooth and of genus 3 then C is a non-hyperelliptic modular curve of genus 3, level N such that

$$J(C)\stackrel{\mathbb{Q}}{\sim} A.$$

OUTPUT: $C: F(X,Y,Z)=0$ or ERROR.

Algorithm (cont.)

Step 3: Compute if there exists

$$F(X,Y,Z) = \sum_{i+j+k=4} a_{ijk} X^i Y^j Z^k \in \mathbb{Q}[X,Y,Z]$$

such that

$$F(x,y,z) = O(q^{c_N}),\ c_N = \frac{4}{3}[SL_2(\mathbb{Z}) : \Gamma_1(N)],$$

Step 4: If $C : F(X,Y,Z) = 0$ is smooth and of genus 3 then C is a non-hyperelliptic modular curve of genus 3, level N such that

$$J(C) \overset{\mathbb{Q}}{\sim} A.$$

OUTPUT: $C : F(X,Y,Z) = 0$ or ERROR.

Algorithm (cont.)

Step 3: Compute if there exists

$$F(X,Y,Z) = \sum_{i+j+k=4} a_{ijk} X^i Y^j Z^k \in \mathbb{Q}[X,Y,Z]$$

such that

$$F(x,y,z) = O(q^{c_N}),\ c_N = \frac{4}{3}[SL_2(\mathbb{Z}) : \Gamma_1(N)],$$

Step 4: If $C : F(X,Y,Z) = 0$ is smooth and of genus 3 then C is a non-hyperelliptic modular curve of genus 3, level N such that

$$J(C) \overset{\mathbb{Q}}{\sim} A.$$

OUTPUT: $C : F(X,Y,Z) = 0$ or ERROR.

Algorithm (cont.)

Step 3: Compute if there exists

$$F(X,Y,Z)=\sum_{i+j+k=4} a_{ijk}X^iY^jZ^k \in \mathbb{Q}[X,Y,Z]$$

such that

$$F(x,y,z)=O(q^{c_N}),\ c_N=\frac{4}{3}[SL_2(\mathbb{Z}):\Gamma_1(N)],$$

Step 4: If $C: F(X,Y,Z)=0$ is smooth and of genus 3 then C is a non-hyperelliptic modular curve of genus 3, level N such that

$$J(C)\overset{\mathbb{Q}}{\sim} A.$$

OUTPUT: $C: F(X,Y,Z)=0$ or ERROR.

Algorithm (cont.)

Step 3: Compute if there exists

$$F(X,Y,Z)=\sum_{i+j+k=4} a_{ijk}X^iY^jZ^k\in\mathbb{Q}[X,Y,Z]$$

such that

$$F(x,y,z)=O(q^{c_N}),\ c_N=\frac{4}{3}[SL_2(\mathbb{Z}):\Gamma_1(N)],$$

Step 4: If $C: F(X,Y,Z)=0$ is smooth and of genus 3 then C is a non-hyperelliptic modular curve of genus 3, level N such that

$$J(C)\overset{\mathbb{Q}}{\sim}A.$$

OUTPUT: $C: F(X,Y,Z)=0$ or ERROR.

Algorithm (cont.)

Step 3: Compute if there exists

$$F(X,Y,Z)=\sum_{i+j+k=4} a_{ijk}X^iY^jZ^k \in \mathbb{Q}[X,Y,Z]$$

such that

$$F(x,y,z)=O(q^{c_N}),\ c_N=\frac{4}{3}[SL_2(\mathbb{Z}):\Gamma_1(N)],$$

Step 4: If $C: F(X,Y,Z)=0$ is smooth and of genus 3 then C is a non-hyperelliptic modular curve of genus 3, level N such that

$$J(C)\overset{\mathbb{Q}}{\sim} A.$$

OUTPUT: $C: F(X,Y,Z)=0$ or ERROR.

Algorithm (cont.)

Step 3: Compute if there exists

$$F(X,Y,Z) = \sum_{i+j+k=4} a_{ijk} X^i Y^j Z^k \in \mathbb{Q}[X,Y,Z]$$

such that

$$F(x,y,z) = O(q^{c_N}),\ c_N = \frac{4}{3}[SL_2(\mathbb{Z}) : \Gamma_1(N)],$$

Step 4: If $C : F(X,Y,Z) = 0$ is smooth and of genus 3 then C is a non-hyperelliptic modular curve of genus 3, level N such that

$$J(C) \overset{\mathbb{Q}}{\sim} A.$$

OUTPUT: $C : F(X,Y,Z) = 0$ or ERROR.

Algorithm (cont.)

Step 3: Compute if there exists

$$F(X,Y,Z) = \sum_{i+j+k=4} a_{ijk} X^i Y^j Z^k \in \mathbb{Q}[X,Y,Z]$$

such that

$$F(x,y,z) = O(q^{c_N}),\ c_N = \frac{4}{3}[SL_2(\mathbb{Z}) : \Gamma_1(N)],$$

Step 4: If $C : F(X,Y,Z) = 0$ is smooth and of genus 3 then C is a non-hyperelliptic modular curve of genus 3, level N such that

$$J(C) \overset{\mathbb{Q}}{\sim} A.$$

OUTPUT: $C : F(X,Y,Z) = 0$ or ERROR.

Algorithm (cont.)

Step 3: Compute if there exists

$$F(X,Y,Z) = \sum_{i+j+k=4} a_{ijk} X^i Y^j Z^k \in \mathbb{Q}[X,Y,Z]$$

such that

$$F(x,y,z) = O(q^{c_N}),\ c_N = \frac{4}{3}[SL_2(\mathbb{Z}) : \Gamma_1(N)],$$

Step 4: If $C : F(X,Y,Z) = 0$ is smooth and of genus 3 then C is a non-hyperelliptic modular curve of genus 3, level N such that

$$J(C) \overset{\mathbb{Q}}{\sim} A.$$

OUTPUT: $C : F(X,Y,Z) = 0$ or ERROR.

Algorithm (cont.)

Step 3: Compute if there exists

$$F(X,Y,Z)=\sum_{i+j+k=4} a_{ijk}X^iY^jZ^k\in\mathbb{Q}[X,Y,Z]$$

such that

$$F(x,y,z)=O(q^{c_N}),\ c_N=\frac{4}{3}[SL_2(\mathbb{Z}):\Gamma_1(N)],$$

Step 4: If $C:F(X,Y,Z)=0$ is smooth and of genus 3 then C is a non-hyperelliptic modular curve of genus 3, level N such that

$$J(C)\overset{\mathbb{Q}}{\sim}A.$$

OUTPUT: $C:F(X,Y,Z)=0$ or ERROR.

```
Magma V2.14-1    Sat Jul 31 2007 15:24:22    [Seed = 1237722563]
Type ? for help.  Type <Ctrl>-D to quit.
> A:=Af(97); A;
[
 Modular symbols space for Gamma_0(97) of weight 2 and dimension 3
  over Rational Field,
 Modular symbols space for Gamma_0(97) of weight 2 and dimension 4
  over Rational Field
]
> NewModularNONHyperellipticCurve(A[1]);
x^3*z - x^2*y^2 - 5*x^2*z^2 + x*y^3 + x*y^2*z + 3*x*y*z^2 +
6*x*z^3 - 3*y^2*z^2 - y*z^3 - 2*z^4;
```

```
Magma V2.14-1    Sat Jul 31 2007 15:24:22    [Seed = 1237722563]
Type ? for help.  Type <Ctrl>-D to quit.

> A:=Af(97); A;
[
 Modular symbols space for Gamma_0(97) of weight 2 and dimension 3
  over Rational Field,
 Modular symbols space for Gamma_0(97) of weight 2 and dimension 4
  over Rational Field
]
> NewModularNONHyperellipticCurve(A[1]);
x^3*z - x^2*y^2 - 5*x^2*z^2 + x*y^3 + x*y^2*z + 3*x*y*z^2 +
6*x*z^3 - 3*y^2*z^2 - y*z^3 - 2*z^4;
```

```
Magma V2.14-1    Sat Jul 31 2007 15:24:22    [Seed = 1237722563]
Type ? for help.  Type <Ctrl>-D to quit.

> A:=Af(97); A;
[
 Modular symbols space for Gamma_0(97) of weight 2 and dimension 3
  over Rational Field,
 Modular symbols space for Gamma_0(97) of weight 2 and dimension 4
  over Rational Field
]
> NewModularNONHyperellipticCurve(A[1]);
x^3*z - x^2*y^2 - 5*x^2*z^2 + x*y^3 + x*y^2*z + 3*x*y*z^2 +
6*x*z^3 - 3*y^2*z^2 - y*z^3 - 2*z^4;
```

```
Magma V2.14-1    Sat Jul 31 2007 15:24:22    [Seed = 1237722563]
Type ? for help.  Type <Ctrl>-D to quit.

> A:=Af(97); A;
[
 Modular symbols space for Gamma_0(97) of weight 2 and dimension 3
  over Rational Field,
 Modular symbols space for Gamma_0(97) of weight 2 and dimension 4
  over Rational Field
]
> NewModularNONHyperellipticCurve(A[1]);
x^3*z - x^2*y^2 - 5*x^2*z^2 + x*y^3 + x*y^2*z + 3*x*y*z^2 +
6*x*z^3 - 3*y^2*z^2 - y*z^3 - 2*z^4;
```

```
Magma V2.14-1    Sat Jul 31 2007 15:24:22    [Seed = 1237722563]
Type ? for help.  Type <Ctrl>-D to quit.
> A:=Af(97); A;
[
 Modular symbols space for Gamma_0(97) of weight 2 and dimension 3
  over Rational Field,
 Modular symbols space for Gamma_0(97) of weight 2 and dimension 4
  over Rational Field
]
> NewModularNONHyperellipticCurve(A[1]);
x^3*z - x^2*y^2 - 5*x^2*z^2 + x*y^3 + x*y^2*z + 3*x*y*z^2 +
6*x*z^3 - 3*y^2*z^2 - y*z^3 - 2*z^4;
```

```
> NewModularNONHyperellipticCurves;

Intrinsic 'NewModularNONHyperellipticCurves'

Signatures:

    (<RngIntElt> N) -> SeqEnum
    [
        prec,
        verbose,
        gamma,
        fixgenus,
        genus,
        check
    ]

All the New Modular NON Hyperelliptic Curve of level N.
```

```
> NewModularNONHyperellipticCurves;

Intrinsic 'NewModularNONHyperellipticCurves'

Signatures:

    (<RngIntElt> N) -> SeqEnum
    [
        prec,
        verbose,
        gamma,
        fixgenus,
        genus,
        check
    ]

All the New Modular NON Hyperelliptic Curve of level N.
```

```
> NewModularNONHyperellipticCurves(17);

     Candidates:=2

     Genus <= 5

  1  ---------------> NO NewMNonHC of level 17 for : <<[ 2 ], 1>>;
[(2)]---------------> NewMNonHC[17,<<[ 0 ], 1>, <[ 2 ], 1>>](<1,
4>):=[ y*u - z*t + z*u + t^2 - 2*t*u - 2*u^2, y*t - z^2 + 5*z*u -
t^2 - 5*t*u - 2*u^2, x^2 - y^2 + 2*y*z - z^2 + 2*z*t - 4*z*u -
2*t^2 + 8*t*u - 5*u^2 ];


<<17, <<[ 0 ], 1>, <[ 2 ], 1>>, [
    y*u - z*t + z*u + t^2 - 2*t*u - 2*u^2,
    y*t - z^2 + 5*z*u - t^2 - 5*t*u - 2*u^2,
    x^2 - y^2 + 2*y*z - z^2 + 2*z*t - 4*z*u - 2*t^2 + 8*t*u - 5*u^2
], <<1, 4>>>>
```

```
> NewModularNONHyperellipticCurves(17);

     Candidates:=2

     Genus <= 5

  1   ---------------> NO NewMNonHC of level 17 for : <<[ 2 ], 1>>;
[(2)]---------------> NewMNonHC[17,<<[ 0 ], 1>, <[ 2 ], 1>>](<1,
4>):=[ y*u - z*t + z*u + t^2 - 2*t*u - 2*u^2, y*t - z^2 + 5*z*u -
t^2 - 5*t*u - 2*u^2, x^2 - y^2 + 2*y*z - z^2 + 2*z*t - 4*z*u -
2*t^2 + 8*t*u - 5*u^2 ];


<<17, <<[ 0 ], 1>, <[ 2 ], 1>>, [
    y*u - z*t + z*u + t^2 - 2*t*u - 2*u^2,
    y*t - z^2 + 5*z*u - t^2 - 5*t*u - 2*u^2,
    x^2 - y^2 + 2*y*z - z^2 + 2*z*t - 4*z*u - 2*t^2 + 8*t*u - 5*u^2
], <<1, 4>>>>
```

C

$X_1(17)$: $\begin{cases} yu - zt + zu + t^2 - 2tu - 2u^2 = 0 \\ yt - z^2 + 5zu - t^2 - 5tu - 2u^2 = 0 \\ x^2 - y^2 + 2yz - z^2 + 2zt - 4zu - 2t^2 + 8tu - 5u^2 = 0 \end{cases}$ 5

$X_1(20)$: $x^4 - y^4 + 8y^2z^2 - 8yz^3 = 0$ 3

$X_1(21)$: $\begin{cases} 4xz + 3y^2 - 4yz - z^2 - 3t^2 + 18tu - 18u^2 = 0 \\ 4xy + 2y^2 + 2yz - 3z^2 - 6t^2 + 18tu - 9u^2 = 0 \\ x^2 - y^2 + yz - z^2 = 0 \end{cases}$ 5

$X_1(24)$: $\begin{cases} yz - z^2 - tu - u^2 = 0 \\ y^2 - 2z^2 - t^2 - 2u^2 = 0 \\ x^2 - z^2 - t^2 + u^2 = 0 \end{cases}$ 5

$C_{25A_{\{4\}}}$: $\begin{cases} xz - y^2 + yt - 2zt + t^2 = 0 \\ x^2t - xt^2 - y^3 + y^2z - 3yzt + 3yt^2 + z^2t - 2zt^2 + t^3 = 0 \end{cases}$ 4

$X_0(43)$: $x^4 + 2x^2y^2 + 8x^2yz + 16x^2z^2 - 3y^4 + 8y^3z + 16y^2z^2 + 48yz^3 + 64z^4 = 0$ 3

... ...

C

$X_1(17)$	:	$\begin{cases} yu - zt + zu + t^2 - 2tu - 2u^2 = 0 \\ yt - z^2 + 5zu - t^2 - 5tu - 2u^2 = 0 \\ x^2 - y^2 + 2yz - z^2 + 2zt - 4zu - 2t^2 + 8tu - 5u^2 = 0 \end{cases}$	5
$X_1(20)$	:	$x^4 - y^4 + 8y^2z^2 - 8yz^3 = 0$	3
$X_1(21)$	:	$\begin{cases} 4xz + 3y^2 - 4yz - z^2 - 3t^2 + 18tu - 18u^2 = 0 \\ 4xy + 2y^2 + 2yz - 3z^2 - 6t^2 + 18tu - 9u^2 = 0 \\ x^2 - y^2 + yz - z^2 = 0 \end{cases}$	5
$X_1(24)$	:	$\begin{cases} yz - z^2 - tu - u^2 = 0 \\ y^2 - 2z^2 - t^2 - 2u^2 = 0 \\ x^2 - z^2 - t^2 + u^2 = 0 \end{cases}$	5
$C_{25A_{\{4\}}}$	:	$\begin{cases} xz - y^2 + yt - 2zt + t^2 = 0 \\ x^2t - xt^2 - y^3 + y^2z - 3yzt + 3yt^2 + z^2t - 2zt^2 + t^3 = 0 \end{cases}$	4
$X_0(43)$	:	$x^4 + 2x^2y^2 + 8x^2yz + 16x^2z^2 - 3y^4 + 8y^3z + 16y^2z^2 + 48yz^3 + 64z^4 = 0$	3
...		... ...	

C

$X_1(17)$: $\begin{cases} yu - zt + zu + t^2 - 2tu - 2u^2 = 0 \\ yt - z^2 + 5zu - t^2 - 5tu - 2u^2 = 0 \\ x^2 - y^2 + 2yz - z^2 + 2zt - 4zu - 2t^2 + 8tu - 5u^2 = 0 \end{cases}$ 5

$X_1(20)$: $x^4 - y^4 + 8y^2z^2 - 8yz^3 = 0$ 3

$X_1(21)$: $\begin{cases} 4xz + 3y^2 - 4yz - z^2 - 3t^2 + 18tu - 18u^2 = 0 \\ 4xy + 2y^2 + 2yz - 3z^2 - 6t^2 + 18tu - 9u^2 = 0 \\ x^2 - y^2 + yz - z^2 = 0 \end{cases}$ 5

$X_1(24)$: $\begin{cases} yz - z^2 - tu - u^2 = 0 \\ y^2 - 2z^2 - t^2 - 2u^2 = 0 \\ x^2 - z^2 - t^2 + u^2 = 0 \end{cases}$ 5

$C_{25A_{\{4\}}}$: $\begin{cases} xz - y^2 + yt - 2zt + t^2 = 0 \\ x^2t - xt^2 - y^3 + y^2z - 3yzt + 3yt^2 + z^2t - 2zt^2 + t^3 = 0 \end{cases}$ 4

$X_0(43)$: $x^4 + 2x^2y^2 + 8x^2yz + 16x^2z^2 - 3y^4 + 8y^3z + 16y^2z^2 + 48yz^3 + 64z^4 = 0$ 3

... ...

C

$$X_1(17) \quad : \quad \begin{cases} yu - zt + zu + t^2 - 2tu - 2u^2 = 0 \\ yt - z^2 + 5zu - t^2 - 5tu - 2u^2 = 0 \\ x^2 - y^2 + 2yz - z^2 + 2zt - 4zu - 2t^2 + 8tu - 5u^2 = 0 \end{cases} \qquad 5$$

$$X_1(20) \quad : \quad x^4 - y^4 + 8y^2z^2 - 8yz^3 = 0 \qquad 3$$

$$X_1(21) \quad : \quad \begin{cases} 4xz + 3y^2 - 4yz - z^2 - 3t^2 + 18tu - 18u^2 = 0 \\ 4xy + 2y^2 + 2yz - 3z^2 - 6t^2 + 18tu - 9u^2 = 0 \\ x^2 - y^2 + yz - z^2 = 0 \end{cases} \qquad 5$$

$$X_1(24) \quad : \quad \begin{cases} yz - z^2 - tu - u^2 = 0 \\ y^2 - 2z^2 - t^2 - 2u^2 = 0 \\ x^2 - z^2 - t^2 + u^2 = 0 \end{cases} \qquad 5$$

$$C_{25A_{\{4\}}} \quad : \quad \begin{cases} xz - y^2 + yt - 2zt + t^2 = 0 \\ x^2t - xt^2 - y^3 + y^2z - 3yzt + 3yt^2 + z^2t - 2zt^2 + t^3 = 0 \end{cases} \qquad 4$$

$$X_0(43) \quad : \quad x^4 + 2x^2y^2 + 8x^2yz + 16x^2z^2 - 3y^4 + 8y^3z + 16y^2z^2 + 48yz^3 + 64z^4 = 0 \qquad 3$$

$\cdots \quad \cdots \quad \cdots \quad \cdots$

C

$$X_1(17) \quad : \quad \begin{cases} yu - zt + zu + t^2 - 2tu - 2u^2 = 0 \\ yt - z^2 + 5zu - t^2 - 5tu - 2u^2 = 0 \\ x^2 - y^2 + 2yz - z^2 + 2zt - 4zu - 2t^2 + 8tu - 5u^2 = 0 \end{cases} \qquad 5$$

$$X_1(20) \quad : \quad x^4 - y^4 + 8y^2z^2 - 8yz^3 = 0 \qquad 3$$

$$X_1(21) \quad : \quad \begin{cases} 4xz + 3y^2 - 4yz - z^2 - 3t^2 + 18tu - 18u^2 = 0 \\ 4xy + 2y^2 + 2yz - 3z^2 - 6t^2 + 18tu - 9u^2 = 0 \\ x^2 - y^2 + yz - z^2 = 0 \end{cases} \qquad 5$$

$$X_1(24) \quad : \quad \begin{cases} yz - z^2 - tu - u^2 = 0 \\ y^2 - 2z^2 - t^2 - 2u^2 = 0 \\ x^2 - z^2 - t^2 + u^2 = 0 \end{cases} \qquad 5$$

$$C_{25A_{\{4\}}} \quad : \quad \begin{cases} xz - y^2 + yt - 2zt + t^2 = 0 \\ x^2t - xt^2 - y^3 + y^2z - 3yzt + 3yt^2 + z^2t - 2zt^2 + t^3 = 0 \end{cases} \qquad 4$$

$$X_0(43) \quad : \quad x^4 + 2x^2y^2 + 8x^2yz + 16x^2z^2 - 3y^4 + 8y^3z + 16y^2z^2 + 48yz^3 + 64z^4 = 0 \qquad 3$$

… … … …

C

$$X_1(17) \quad : \quad \begin{cases} yu - zt + zu + t^2 - 2tu - 2u^2 = 0 \\ yt - z^2 + 5zu - t^2 - 5tu - 2u^2 = 0 \\ x^2 - y^2 + 2yz - z^2 + 2zt - 4zu - 2t^2 + 8tu - 5u^2 = 0 \end{cases} \qquad 5$$

$$X_1(20) \quad : \quad x^4 - y^4 + 8y^2z^2 - 8yz^3 = 0 \qquad 3$$

$$X_1(21) \quad : \quad \begin{cases} 4xz + 3y^2 - 4yz - z^2 - 3t^2 + 18tu - 18u^2 = 0 \\ 4xy + 2y^2 + 2yz - 3z^2 - 6t^2 + 18tu - 9u^2 = 0 \\ x^2 - y^2 + yz - z^2 = 0 \end{cases} \qquad 5$$

$$X_1(24) \quad : \quad \begin{cases} yz - z^2 - tu - u^2 = 0 \\ y^2 - 2z^2 - t^2 - 2u^2 = 0 \\ x^2 - z^2 - t^2 + u^2 = 0 \end{cases} \qquad 5$$

$$C_{25A_{\{4\}}} \quad : \quad \begin{cases} xz - y^2 + yt - 2zt + t^2 = 0 \\ x^2t - xt^2 - y^3 + y^2z - 3yzt + 3yt^2 + z^2t - 2zt^2 + t^3 = 0 \end{cases} \qquad 4$$

$$X_0(43) \quad : \quad x^4 + 2x^2y^2 + 8x^2yz + 16x^2z^2 - 3y^4 + 8y^3z + 16y^2z^2 + 48yz^3 + 64z^4 = 0 \qquad 3$$

... ...

C			
$X_1(17)$	:	$\begin{cases} yu - zt + zu + t^2 - 2tu - 2u^2 = 0 \\ yt - z^2 + 5zu - t^2 - 5tu - 2u^2 = 0 \\ x^2 - y^2 + 2yz - z^2 + 2zt - 4zu - 2t^2 + 8tu - 5u^2 = 0 \end{cases}$	5
$X_1(20)$	:	$x^4 - y^4 + 8y^2z^2 - 8yz^3 = 0$	3
$X_1(21)$	:	$\begin{cases} 4xz + 3y^2 - 4yz - z^2 - 3t^2 + 18tu - 18u^2 = 0 \\ 4xy + 2y^2 + 2yz - 3z^2 - 6t^2 + 18tu - 9u^2 = 0 \\ x^2 - y^2 + yz - z^2 = 0 \end{cases}$	5
$X_1(24)$	:	$\begin{cases} yz - z^2 - tu - u^2 = 0 \\ y^2 - 2z^2 - t^2 - 2u^2 = 0 \\ x^2 - z^2 - t^2 + u^2 = 0 \end{cases}$	5
$C_{25A_{\{4\}}}$	:	$\begin{cases} xz - y^2 + yt - 2zt + t^2 = 0 \\ x^2t - xt^2 - y^3 + y^2z - 3yzt + 3yt^2 + z^2t - 2zt^2 + t^3 = 0 \end{cases}$	4
$X_0(43)$	:	$x^4 + 2x^2y^2 + 8x^2yz + 16x^2z^2 - 3y^4 + 8y^3z + 16y^2z^2 + 48yz^3 + 64z^4 = 0$	3
...		... ...	

C			
$X_1(17)$	:	$\left\{\begin{array}{l} yu - zt + zu + t^2 - 2tu - 2u^2 = 0 \\ yt - z^2 + 5zu - t^2 - 5tu - 2u^2 = 0 \\ x^2 - y^2 + 2yz - z^2 + 2zt - 4zu - 2t^2 + 8tu - 5u^2 = 0 \end{array}\right.$	5
$X_1(20)$	:	$x^4 - y^4 + 8y^2z^2 - 8yz^3 = 0$	3
$X_1(21)$	:	$\left\{\begin{array}{l} 4xz + 3y^2 - 4yz - z^2 - 3t^2 + 18tu - 18u^2 = 0 \\ 4xy + 2y^2 + 2yz - 3z^2 - 6t^2 + 18tu - 9u^2 = 0 \\ x^2 - y^2 + yz - z^2 = 0 \end{array}\right.$	5
$X_1(24)$	:	$\left\{\begin{array}{l} yz - z^2 - tu - u^2 = 0 \\ y^2 - 2z^2 - t^2 - 2u^2 = 0 \\ x^2 - z^2 - t^2 + u^2 = 0 \end{array}\right.$	5
$C_{25A_{\{4\}}}$	:	$\left\{\begin{array}{l} xz - y^2 + yt - 2zt + t^2 = 0 \\ x^2t - xt^2 - y^3 + y^2z - 3yzt + 3yt^2 + z^2t - 2zt^2 + t^3 = 0 \end{array}\right.$	4
$X_0(43)$	:	$x^4 + 2x^2y^2 + 8x^2yz + 16x^2z^2 - 3y^4 + 8y^3z + 16y^2z^2 + 48yz^3 + 64z^4 = 0$	3
$\ldots$		$\ldots \quad \ldots \quad \ldots$	

Final Remark

- k a field, $char(k) = 0$,
- $C_{/k}$ a curve,
- $P \in C(k)$
- q an analytic uniformizing parameter in $\hat{O}_{C,P}$,
- $H^0(C, \Omega^1_C) = \langle \omega_1, \ldots, \omega_g \rangle_{\mathbb{C}}$.

General Algorithm

INPUT: $(C, q, \{\omega_1, \ldots, \omega_g\})$.
Step 1: Write $\omega_i = w_i dq$ with $w_i \in k[[q]], \forall i = 1, \ldots, g$.
Step 2: Determine if C is hyperelliptic or not.
OUTPUT: A model for $C_{/k}$.

Final Remark

- k a field, $char(k) = 0$,
- $C_{/k}$ a curve,
- $P \in C(k)$
- q an analytic uniformizing parameter in $\hat{O}_{C,P}$,
- $H^0(C, \Omega^1_C) = \langle \omega_1, \ldots, \omega_g \rangle_{\mathbb{C}}$.

General Algorithm

INPUT: $(C, q, \{\omega_1, \ldots, \omega_g\})$.

Step 1: Write $\omega_i = w_i dq$ with $w_i \in k[[q]], \forall i = 1, \ldots, g$.

Step 2: Determine if C is hyperelliptic or not.

OUTPUT: A model for $C_{/k}$.

Final Remark

- k a field, $char(k) = 0$,
- $C_{/k}$ a curve,
- $P \in C(k)$
- q an analytic uniformizing parameter in $\hat{O}_{C,P}$,
- $H^0(C, \Omega^1_C) = \langle \omega_1, \ldots, \omega_g \rangle_{\mathbb{C}}$.

General Algorithm

INPUT: $(C, q, \{\omega_1, \ldots, \omega_g\})$.

Step 1: Write $\omega_i = w_i dq$ with $w_i \in k[[q]], \forall i = 1, \ldots, g$.

Step 2: Determine if C is hyperelliptic or not.

OUTPUT: A model for $C_{/k}$.

Final Remark

- k a field, $char(k) = 0$,
- $C_{/k}$ a curve,
- $P \in C(k)$
- q an analytic uniformizing parameter in $\hat{O}_{C,P}$,
- $H^0(C, \Omega^1_C) = \langle \omega_1, \ldots, \omega_g \rangle_{\mathbb{C}}$.

General Algorithm

INPUT: $(C, q, \{\omega_1, \ldots, \omega_g\})$.

Step 1: Write $\omega_i = w_i dq$ with $w_i \in k[[q]]$, $\forall i = 1, \ldots, g$.

Step 2: Determine if C is hyperelliptic or not.

OUTPUT: A model for $C_{/k}$.

Final Remark

- k a field, $char(k) = 0$,
- $C_{/k}$ a curve,
- $P \in C(k)$
- q an analytic uniformizing parameter in $\hat{O}_{C,P}$,
- $H^0(C, \Omega^1_C) = \langle \omega_1, \ldots, \omega_g \rangle_{\mathbb{C}}$.

General Algorithm

INPUT: $(C, q, \{\omega_1, \ldots, \omega_g\})$.

Step 1: Write $\omega_i = w_i dq$ with $w_i \in k[[q]]$, $\forall i = 1, \ldots, g$.

Step 2: Determine if C is hyperelliptic or not.

OUTPUT: A model for $C_{/k}$.

Final Remark

- k a field, $char(k) = 0$,
- $C_{/k}$ a curve,
- $P \in C(k)$
- q an analytic uniformizing parameter in $\hat{O}_{C,P}$,
- $H^0(C, \Omega^1_C) = \langle \omega_1, \ldots, \omega_g \rangle_{\mathbb{C}}$.

General Algorithm

INPUT: $(C, q, \{\omega_1, \ldots, \omega_g\})$.

Step 1: Write $\omega_i = w_i dq$ with $w_i \in k[[q]]$, $\forall i = 1, \ldots, g$.

Step 2: Determine if C is hyperelliptic or not.

OUTPUT: A model for $C_{/k}$.

Final Remark

- k a field, $char(k) = 0$,
- $C_{/k}$ a curve,
- $P \in C(k)$
- q an analytic uniformizing parameter in $\hat{O}_{C,P}$,
- $H^0(C, \Omega^1_C) = \langle \omega_1, \ldots, \omega_g \rangle_{\mathbb{C}}$.

General Algorithm

INPUT: $(C, q, \{\omega_1, \ldots, \omega_g\})$.

Step 1: Write $\omega_i = w_i dq$ with $w_i \in k[[q]], \forall i = 1, \ldots, g$.

Step 2: Determine if C is hyperelliptic or not.

OUTPUT: A model for $C_{/k}$.

Final Remark

- k a field, $char(k) = 0$,
- $C_{/k}$ a curve,
- $P \in C(k)$
- q an analytic uniformizing parameter in $\hat{O}_{C,P}$,
- $H^0(C, \Omega^1_C) = \langle \omega_1, \ldots, \omega_g \rangle_{\mathbb{C}}$.

General Algorithm

INPUT: $(C, q, \{\omega_1, \ldots, \omega_g\})$.

Step 1: Write $\omega_i = w_i dq$ with $w_i \in k[[q]], \forall i = 1, \ldots, g$.

Step 2: Determine if C is hyperelliptic or not.

OUTPUT: A model for $C_{/k}$.

Final Remark

- k a field, $char(k) = 0$,
- $C_{/k}$ a curve,
- $P \in C(k)$
- q an analytic uniformizing parameter in $\hat{O}_{C,P}$,
- $H^0(C, \Omega^1_C) = \langle \omega_1, \dots, \omega_g \rangle_{\mathbb{C}}$.

General Algorithm

INPUT: $(C, q, \{\omega_1, \dots, \omega_g\})$.

Step 1: Write $\omega_i = w_i dq$ with $w_i \in k[[q]], \forall i = 1, \dots, g$.

Step 2: Determine if C is hyperelliptic or not.

OUTPUT: A model for $C_{/k}$.

Final Remark

- k a field, $char(k) = 0$,
- $C_{/k}$ a curve,
- $P \in C(k)$
- q an analytic uniformizing parameter in $\hat{O}_{C,P}$,
- $H^0(C, \Omega^1_C) = \langle \omega_1, \ldots, \omega_g \rangle_{\mathbb{C}}$.

General Algorithm

INPUT: $(C, q, \{\omega_1, \ldots, \omega_g\})$.

Step 1: Write $\omega_i = w_i dq$ with $w_i \in k[[q]], \forall i = 1, \ldots, g$.

Step 2: Determine if C is hyperelliptic or not.

OUTPUT: A model for $C_{/k}$.

Final Remark

- k a field, $char(k) = 0$,
- $C_{/k}$ a curve,
- $P \in C(k)$
- q an analytic uniformizing parameter in $\hat{O}_{C,P}$,
- $H^0(C, \Omega^1_C) = \langle \omega_1, \ldots, \omega_g \rangle_{\mathbb{C}}$.

General Algorithm

INPUT: $(C, q, \{\omega_1, \ldots, \omega_g\})$.
Step 1: Write $\omega_i = w_i dq$ with $w_i \in k[[q]]$, $\forall i = 1, \ldots, g$.
Step 2: Determine if C is hyperelliptic or not.
OUTPUT: A model for $C_{/k}$.