Minimizing Representations over Number Fields

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Abstract

Finding minimal fields of definition for representations is one of the most important unsolved problems of computational representation theory. While good methods exist for representations over finite fields, it is still an open question in the case of number fields. In this paper we give a practical method for finding minimal defining subfields for absolutely irreducible representations. We illustrate the new algorithm by determining a minimal field for each absolutely irreducible representation of Sz(8).

Key words: minimizing group representations, meat-axe

1 Introduction

Let us fix a group G and an absolutely irreducible representation

$$\rho : G \to \mathrm{GL}(n, K)$$

over some number field K.

In the case when G is finite and solvable, there are well known algorithms [1] for constructing absolutely irreducible representations. Typically these methods construct a suitable cyclotomic field containing "enough" roots of unity. However, the field used to compute representations is in general, too large. These methods are available as part of systems such as Magma [2].

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