MINIMAL DISCRIMINANTS FOR FIELDS WITH SMALL FROBENIUS GROUPS AS GALOIS GROUPS

CLAUS FIEKER AND JÜRGEN KLÜNERS

Abstract. We apply class field theory to the computation of the minimal discriminants for certain solvable groups. In particular we apply our techniques to small Frobenius groups and all imprimitive degree 8 groups such that the corresponding fields have only a degree 2 and no degree 4 subfield.

1. Introduction

There is a long tradition in number theory to compile tables of number fields matching certain criteria. Commonly one computes tables of fields of a given degree or a specific Galois group that are complete with respect to some bound on the discriminant. So far, most of the tables were build with the help of geometric methods based on a theorem of Hunter [**?**, Thm 9.3.1] which states the existence of primitive elements that are not too large in comparison to the discriminant.
Recently the advent of constructive methods in class field theory [**?**, **?**] made it feasible to build large tables with the help of class field theory rather than using the geometric methods. Of course, this applies mainly to the construction of fields with solvable Galois group. For example, in [**?**] class field theory is used to compute the minimal discriminants for all octic fields containing a quartic subfield.
In this paper we illustrate the use of class field theory to construct tables of fields where the Galois group is a small solvable group or a Frobenius group. In particular we obtain the minimal discriminants for octic fields having only a degree 2 subfield and no degree 4 one. This is done by an analysis of the relative Galois group over the degree 2 field. Since the only possibilities here are $\mathfrak{A}_4$ and $\mathfrak{S}_4$ we are in the situation of solvable groups.
As a further application we construct the minimal fields with Galois group isomorphic to $C_p \rtimes C_l$ for $p \in \{7, 11, 13\}$ and all $1 < l \mid (p-1)$ and two primitive solvable groups in degree 8.

2. Notations

Let K/k be a finite field extension. By $d_{K/k}$ we denote the relative discriminant of the ring of integers $\mathbb{Z}_K$ of K as an ideal of k. In addition, d_K always denotes the absolute discriminant (so $d_{K/\mathbb{Q}} = d_K\mathbb{Z}$). We say that K/k has Galois group G, or short that K is a G-field, if the normal closure N of K/k has Galois group G over k. $\mathrm{N}_{K/k}$ denotes the norm function extended to ideals. Since the fractional ideals of the ring of integers of any number field form a group that is freely generated by

Part of this article was written during a visit by the second author to the Computational Algebra Group at the University of Sydney in September, 2001.