# Recent Developments in KASH

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# Overview

**KANT** Computational Algebraic Number Theory
(C-Library)

**KASH** KANT Shell
(GAP 3 with extensions)

**QaoS** Query algebraic objects System
(Database)

**GiANT** Graphical Algebraic Number Theory
(Graphical User Interface)

# History

**1987** KANT V1 (Fortran Library)

**1992** KANT V2 (C Library), built on the Cayley platform

**1994** KANT V4 (C Library), built on the Magma platform

**1995** KASH 1.0 (KANT Shell), based on GAP 3

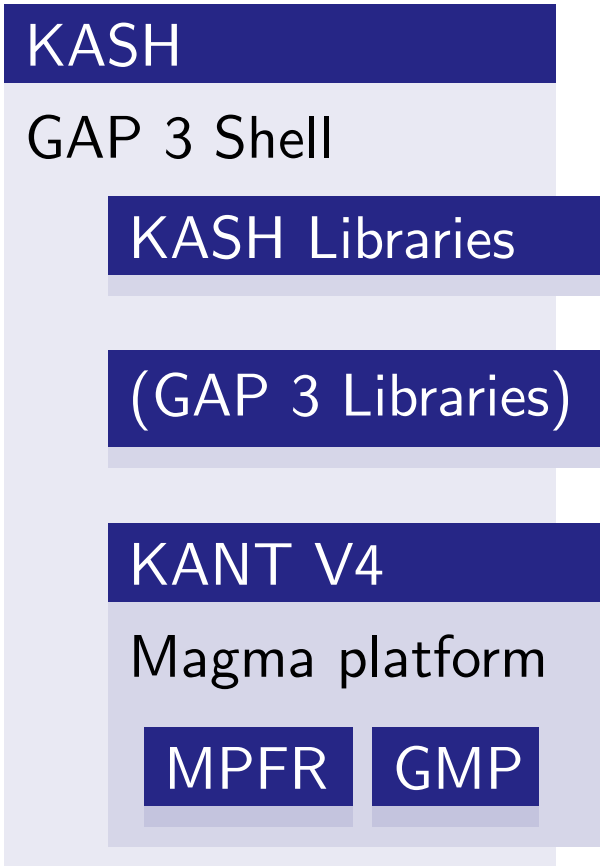
**1996** KASH 1.7, Database for number fields

**1999** KASH 2.1

**2004** KASH 2.4, WWW Database, webkash

**2005** KASH 3, graphical user interface GiANT, QaoS Databases

# Architecture



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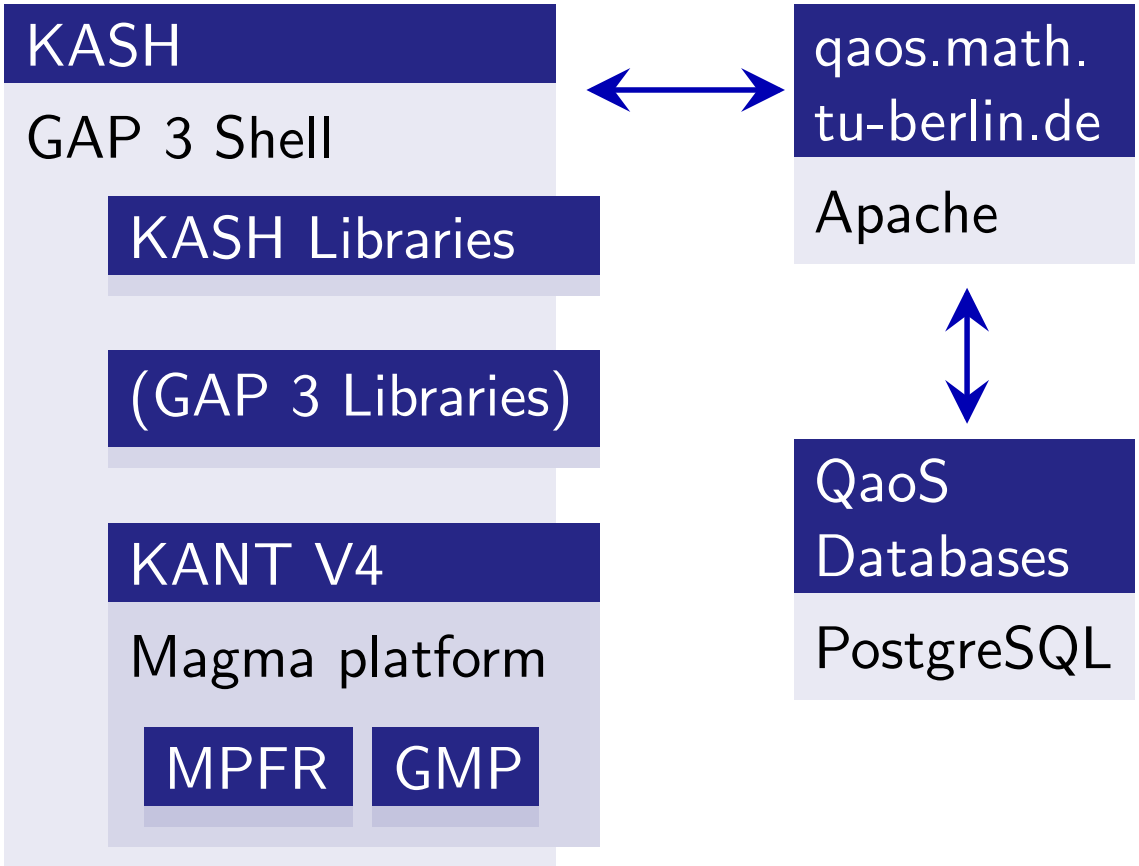

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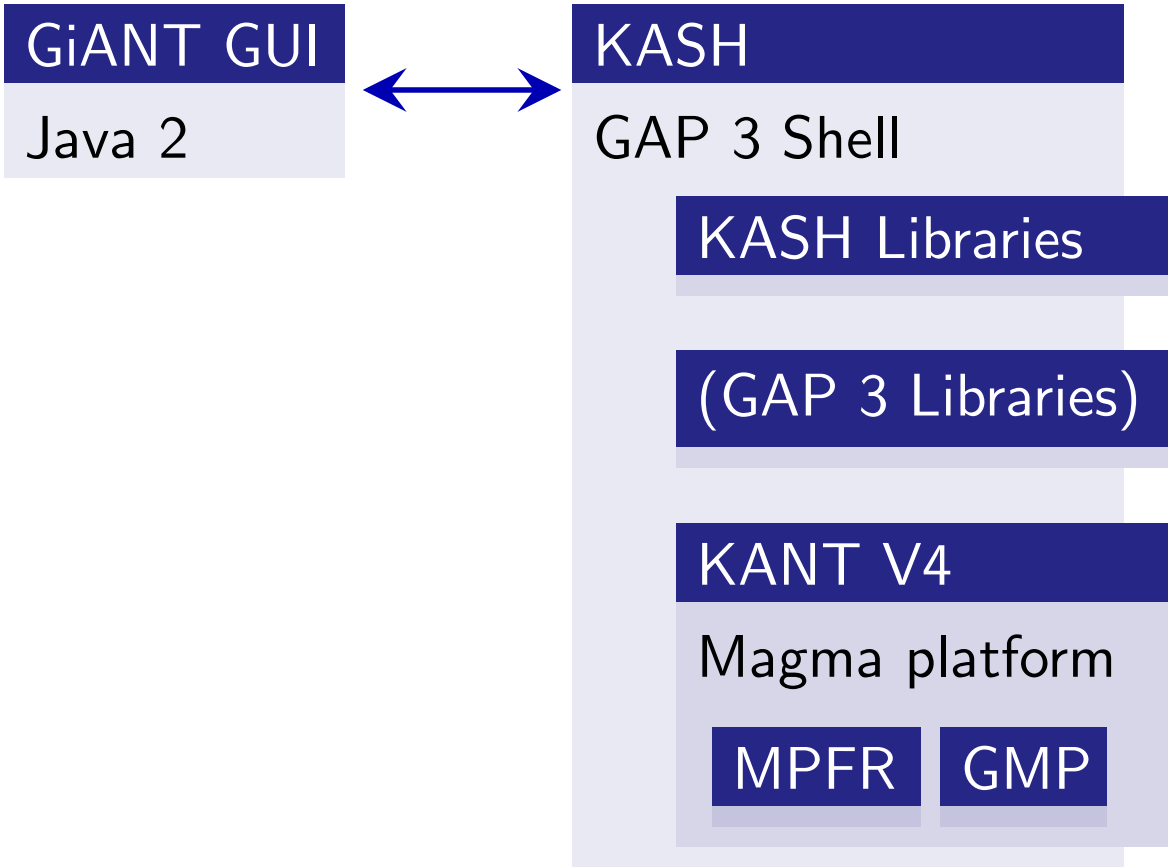

# KASH 3 – Features

## Number Fields

- Integral Bases
- Galois Groups up to Degree 23
- Unit Groups
- Class Groups
- Class Fields

# KASH 3 – Features

## Number Fields

## Function Fields

- Finite and Infinite Maximal Orders
- Unit Groups
- Riemann Roch Spaces
- Divisor Class Groups

# KASH 3 – Features

## Number Fields

## Function Fields

## Local Fields

- p-adic Fields
- Laurent Series
- Polynomial Factorization
- [Unit Groups]

# KASH 3 – Features

## Number Fields

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## Local Fields

## Diophantine Equations

- Norm Equations
- Thue Equations
- Unit Equations

# KASH 3 Help System

## Features

- Inline documentation
- Represented as KASH records
- Help system written in the shell
- Handbooks and tutorials are compiled from the same source (Inline Documentation)
- HTML and print version via XML output
- Inline help with complex search patterns

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### Search Patterns

- Full Text (?*)
- Name (?)
- Substring of Name (?!)
- Input Signature (?(*type*))
- Output Types (?->*type*)

# Documentation Entries

## Kinds of Documentation Entries

- Keyword
- Type
- Function
- Constant
- Statement
- Reference

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## Documentation Record Entries

- Name, Signature
- Examples
- Cross References
- Authors
- Literature References

# KASH 3 Help Example

```
kash% ?!Type

1108: DecompositionType(<ord^num> 0, <elt-ord^rat> p) -> <seq()>
3243: Types
3277: NewType(<string> newtype, <elt-ord^rat> level) -> <type>
3279: ShowTypes()
3280: Type(<any> obj) -> <type>
3299: type

kash% ?3280
```

# KASH 3 Help Example

```
kash% ?3280

#ab84be: FUNCTION (3280)
Type(<any> obj) -> <type>

PURPOSE:
Returns the type of 'obj'.  If 'obj' is extended, i.e., it is a
record with a 'base' component, then the type of the base component
is returned.  If 'obj' is a record with a 'type' component 'obj.type'
is returned.

EXAMPLE:
Type(3);

SEE ALSO:
3243: Types
3277: NewType(<string> newtype, <elt-ord^rat> level) -> <type>
3450: record
```

# KASH 3 – Types

## Goals and Constraints

- Matching of Datatypes from C-Library
- Representation of Categories
- Easy Type Matching for Overloading
- User Defined Types

# KASH 3 Type System

## Modular Types

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### Examples

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|---|---|
| atoms for *algebraic structure* | `alg`, `fld`, `ord`, ... |
| atoms for *specifier* | `fun`, `num`, `pol`, ... |

# KASH 3 Type System

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[elt–] ⟨*algebraic structure*⟩^⟨*specifier*⟩[ / ⟨...⟩]

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| atoms for *specifier* | `fun`, `num`, `pol`, ... |

| | |
|---|---|
| `fld^fun` | Type of Function Fields |
| `ord^num` | Type of Orders of Number Fields |
| `elt-ord^num` | Type of Algebraic Integers |
| `alg^pol/fld^num` | Type of Polynomial Algebras over Number Fields |

# KASH 3 – Methods

## Type Matching

| | |
|---|---|
| `any` | matches any atom |
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## Methods

`InstallMethod`(*documentation record*, *function*)

installs *function* as a method with name and signature given by the *documentation record*.

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## Other Special Record Entries

| | |
|---|---|
| `.`*n* | *n*-th generator of a structure |
| `.type` | sets the type of a record |
| `.operations` | (as in GAP) for overloading of: `+`, `-`, `*`, `/`, `^`, `in`, `mod`, `Print` |
| `.ext`*n* | for multiple return values. |

```
InstallMethod(
 rec(
  name := "Map",
  kind := "FUNCTION",
  sin  := [[any,"D"],[any,"C"],[func,"phi"]],
  sou  := [[map()]],
  short:= "Create a map with domain 'D' and codomain 'C' "
           +"from the function 'phi'.  ",
  see  := [DocHash("Composition(map(),map())")]
 ),
 function(domain,codomain,phi)
  return rec( base := phi,
      domain := domain,
      codomain := codomain,
      type := map(Type(domain),Type(codomain)),
      operations := rec(Print:= __Print_map, \*:=Composition)
      );
 end
);
```

```
InstallMethod(
 rec(
  name := "Composition",
  kind := "FUNCTION",
  sin  := [[map(),"phi"],[map(),"psi"]],
  sou  := [[map()]],
  short:= "The composition 'phi*psi' of the maps 'phi' and 'psi'.",
  ex   := ["add5 := Map(Z,Z,function(a) return a+5; end);"+
     "add5; add5(2); add10 := Composition(add5,add5); add10(2);"],
  see  := [DocHash("Map(any,any,func)")]
 ),
 function(phi,psi)
  return rec( base := function(a) return phi(psi(a)); end,
      domain := Domain(psi),
      codomain := Codomain(phi),
      type := map(Type(Domain(psi)),Type(Codomain(phi))),
      operations := rec(Print := __Print_map, \*:=Composition)
      );
 end
);
```

```
kash% add5 := Map(Z,Z,function(a) return a+5; end);
Mapping from ord^rat: Z to ord^rat: Z

kash% add5(2);
7

kash% add10 := add5*add5;
Mapping from ord^rat: Z to ord^rat: Z

kash% add10(2);
12
```

# GiANT – Graphical Algorithmic Number Theory

written by Aneesh Karve

## Evolution of Computer Algebra Systems

| | | | | |
|---|---|---|---|---|
| Libraries | $\longrightarrow$ | Shells | $\longrightarrow$ | Graphical User Interfaces |
| KANT | $\longrightarrow$ | KASH | $\longrightarrow$ | GiANT |

## GiANT is a Desktop Environment for working with Number Fields

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## GiANT is a Desktop Environment for working with Number Fields

- Desktop shows Towers of Field Extensions
- Moving of Elements, Ideals, and Polynomials by Drag & Drop
- Arithmetic with Elements, Ideals, and Polynomials
- Computation of Invariants
- Architecture: KASH 2.5, Java 2

## KASH can be downloaded from:

`http://www.math.tu-berlin.de/~kant`

## GiANT is released under the GPL, see:

`http://giantsystem.sourceforge.net`

## Thank You