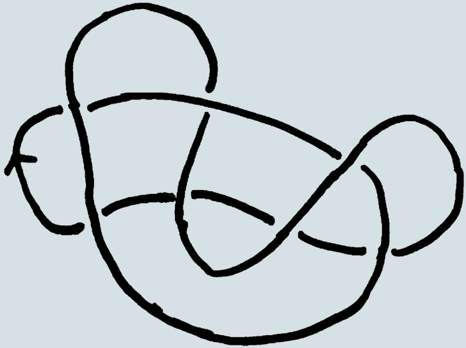
Some notes on the Vogel Algorithm

Dan Roozemond

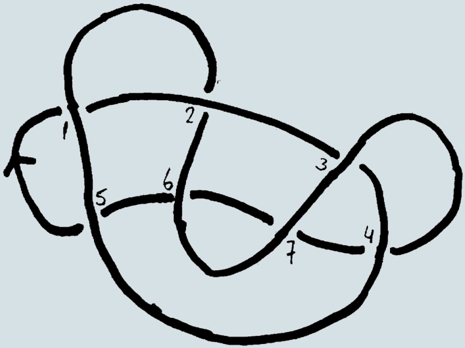
6 June 2007

Contents

Knots



Representing a Knot: The Gauss Code



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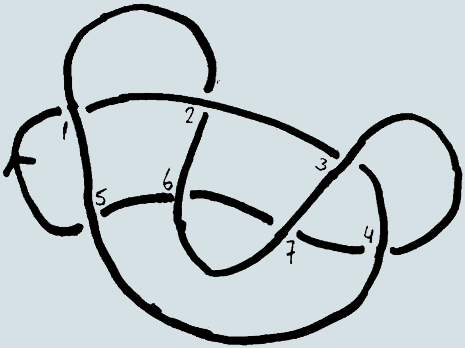
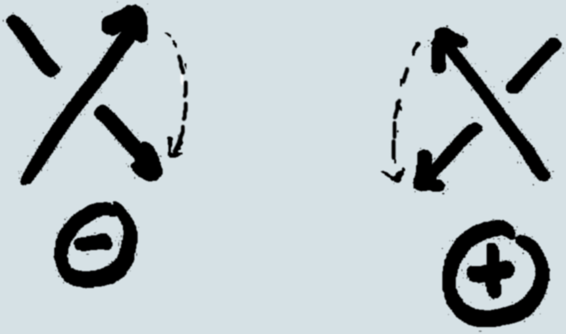


Gauss Code: -1, +2, -3, +4, +5, +1, -2, +6, +7, +3, -4, -7, -6, -5

Representing a Knot: Oriented Gauss Code



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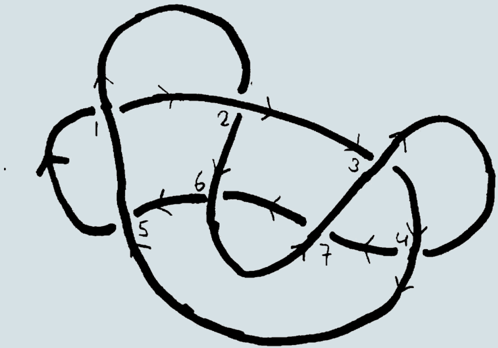

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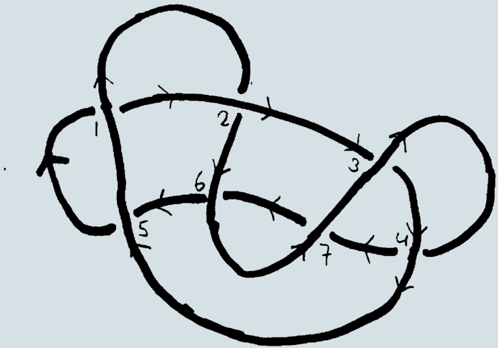
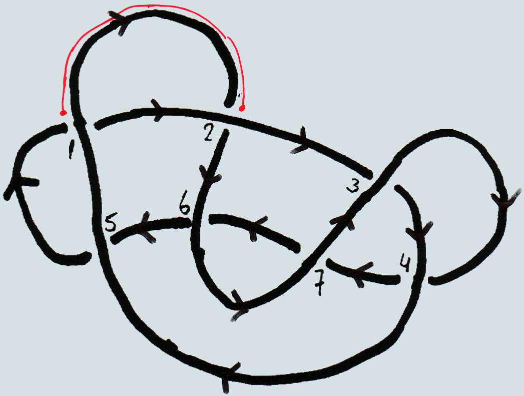


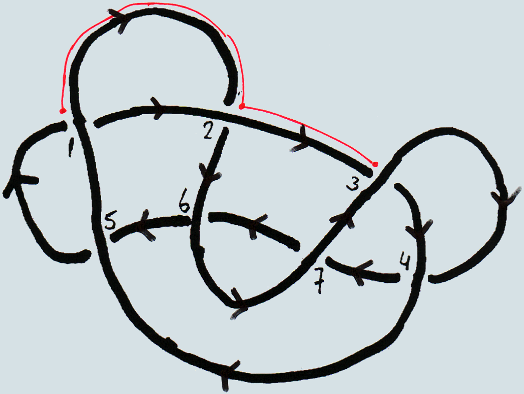
Oriented Gauss Code:
-1, +2, -3, +4, +5, +1, -2, +6, +7, +3, -4, -7, -6, -5 / - - - - + - +

Seifert Circles



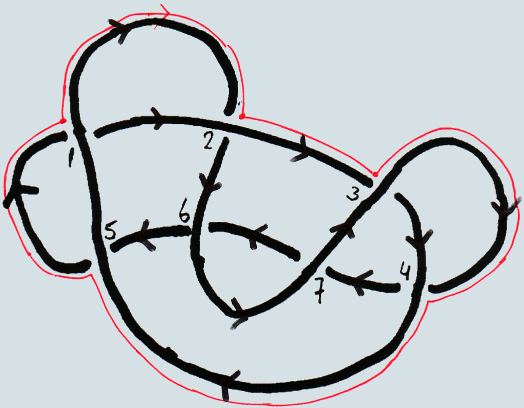
Seifert Circles: [1,2,

Seifert Circles



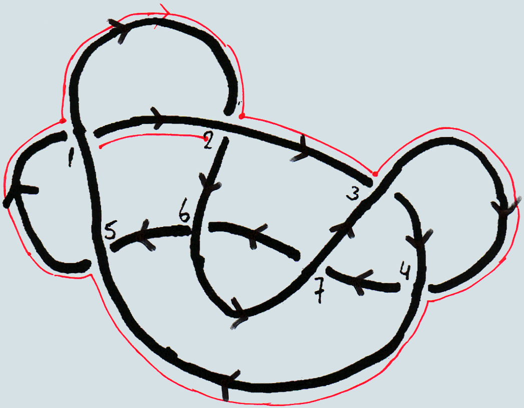
Seifert Circles: [1,2,3,

Seifert Circles



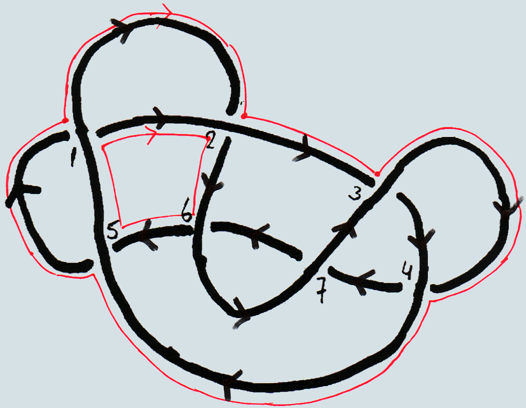
Seifert Circles: [1,2,3,4,5

Seifert Circles



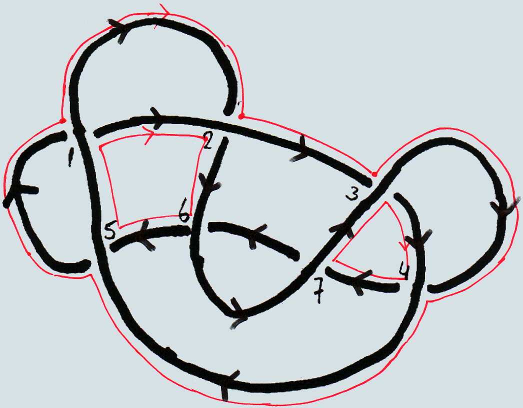
Seifert Circles: [1,2,3,4,5],[1,2

Seifert Circles



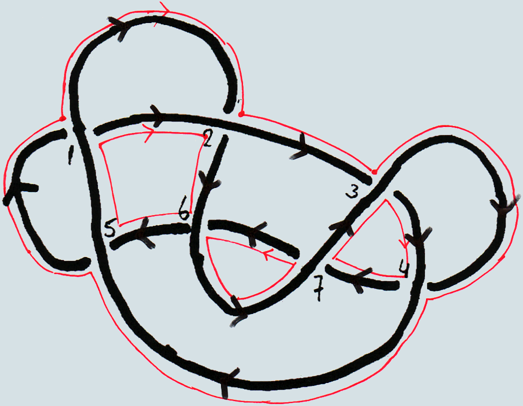
Seifert Circles: [1,2,3,4,5],[1,2,6,5]

Seifert Circles



Seifert Circles: [1,2,3,4,5],[1,2,6,5],[3,4,7]

Seifert Circles



Seifert Circles: [1,2,3,4,5],[1,2,6,5],[3,4,7],[6,7]

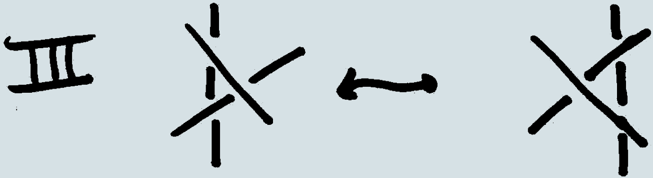
Reidemeister Moves - I

$$\ldots, a, +N, -N, b, \ldots \leftrightarrow \ldots, a, b, \ldots$$

Reidemeister Moves - II

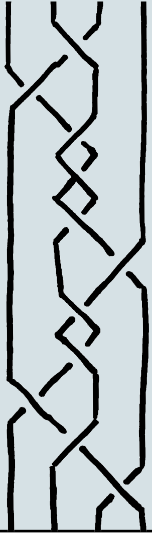
$$\ldots, a, b, \ldots, c, d, \ldots \leftrightarrow \ldots, a, N, M, b, \ldots, c, -M, -N, d, \ldots$$

Reidemeister Moves - III

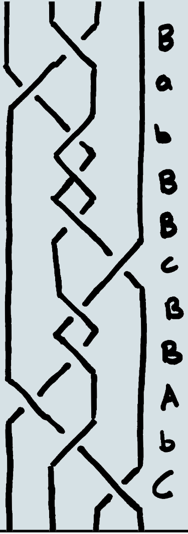


$$\dots, +a, +b, \dots, -b, +c, \dots, -c, -a, \dots$$
$$\leftrightarrow$$
$$\dots, +b, +c, \dots, +a, -b, \dots, -a, -c, \dots$$

Braids

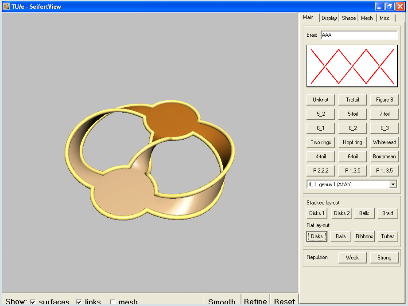


Braids

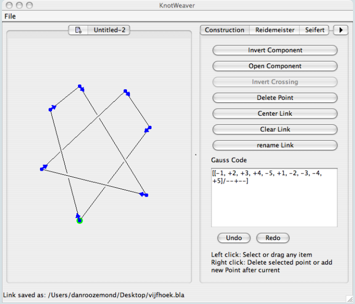


Seifertview / Knotweaver

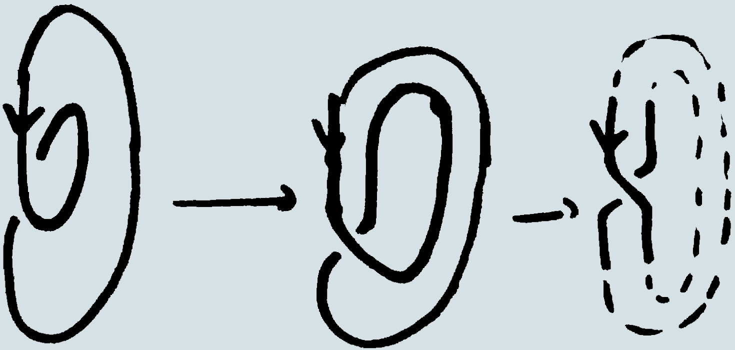
[vanWijk, 2005]



[Vos, 2007]



Knots $\leftrightarrow$ Braids

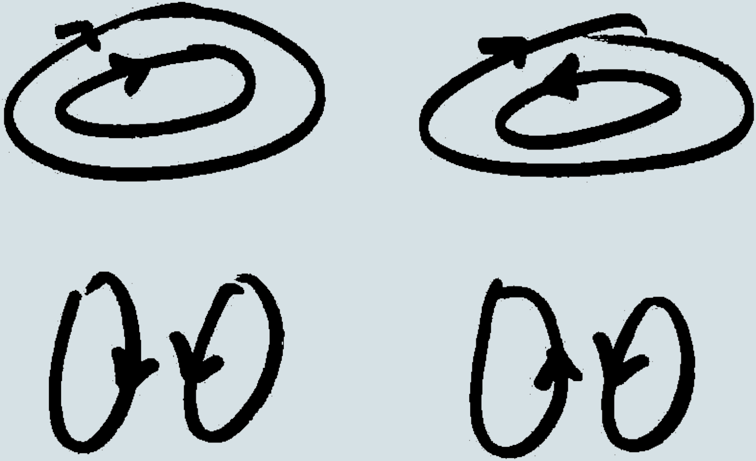


Vogel Moves

[Yamada, 1987], [Vogel, 1990]

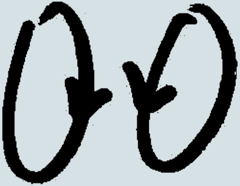
All incoherently oriented Seifert circles can be removed by carrying out Vogel moves. Once this is done, the braid word can easily be read off.

Incoheriently Oriented?



Incoheriently Oriented?

OK



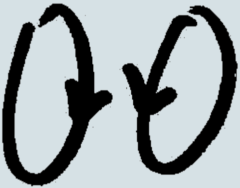
Incoheriently Oriented?

OK NOT OK

Incoheriently Oriented?

OK

NOT OK

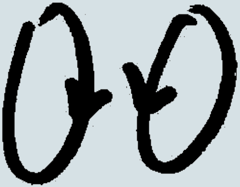


OK

Incoheriently Oriented?

OK

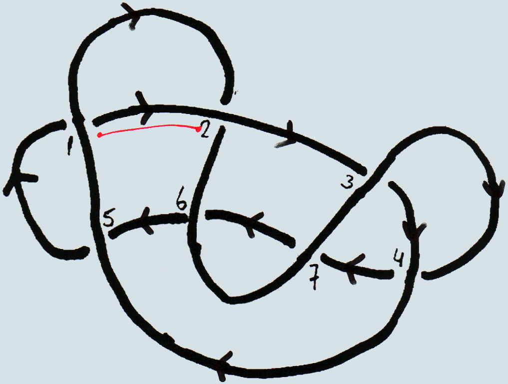
NOT OK



OK

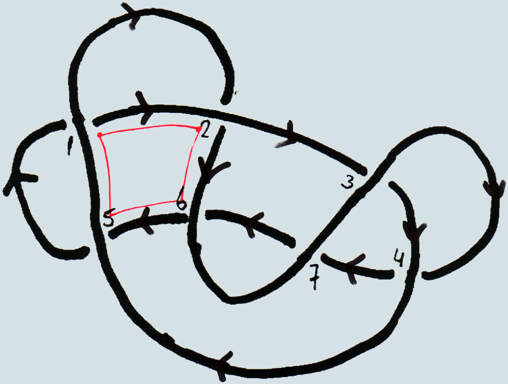
NOT OK

Finding Incoherently Oriented Seifert Circles: Faces



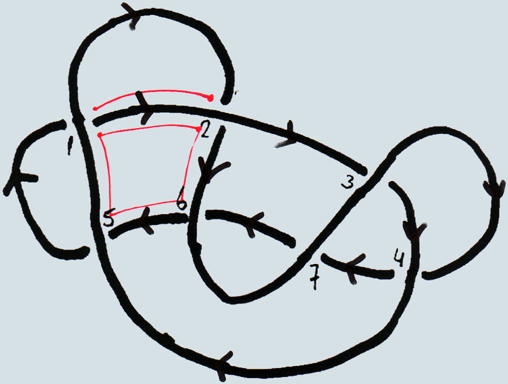
Faces: [R1,2,

Finding Incoherently Oriented Seifert Circles: Faces



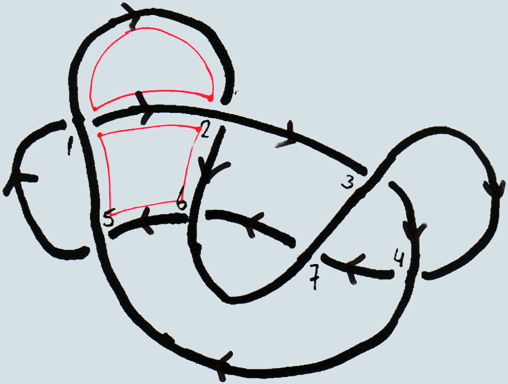
Faces: [R1,2,6,5]

Finding Incoherently Oriented Seifert Circles: Faces



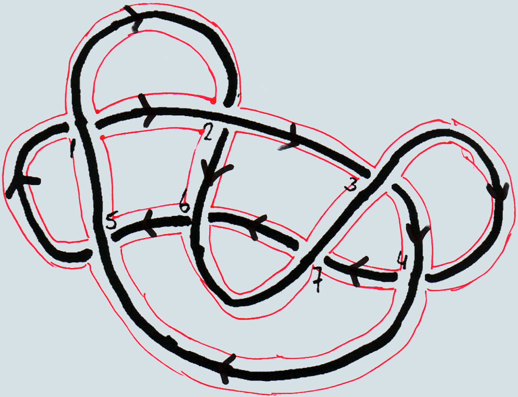
Faces: [R1,2,6,5],[L1,2,

Finding Incoherently Oriented Seifert Circles: Faces



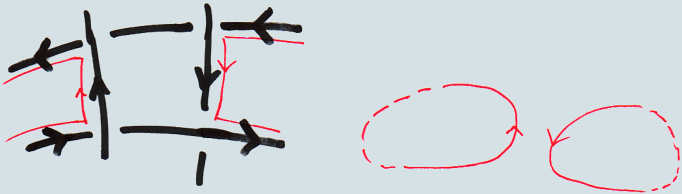
Faces: [R1,2,6,5],[L1,2]

Finding Incoherently Oriented Seifert Circles: Faces



Faces: [R1,2,6,5],[L1,2],[R2,3,7,6],[L2,3,4,5,1],. . .,[R5,1]

The Vogel Move



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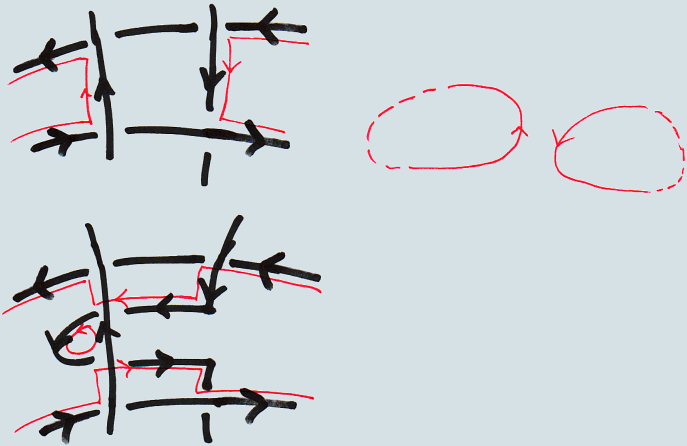

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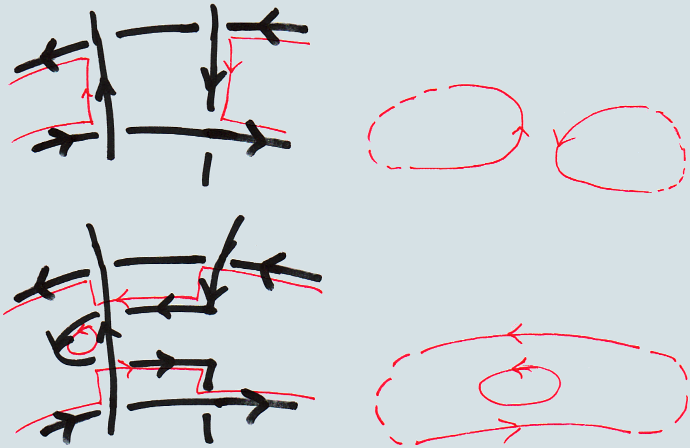
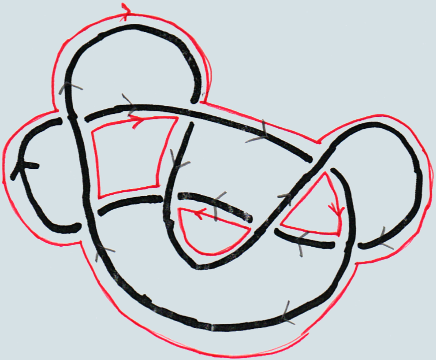
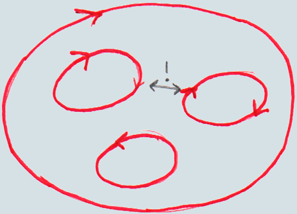


Vogel Moves in Practice





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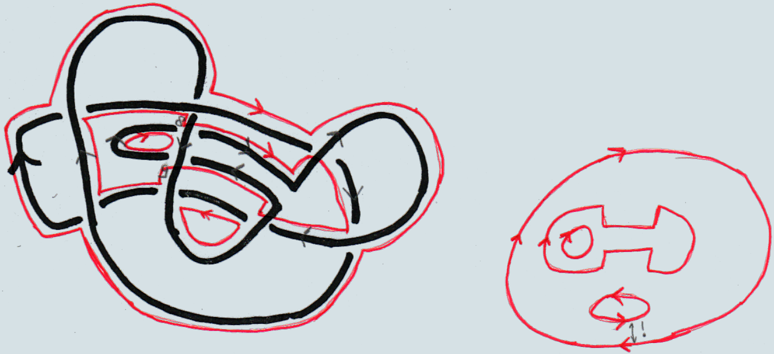

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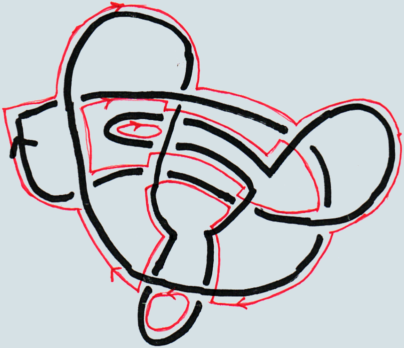

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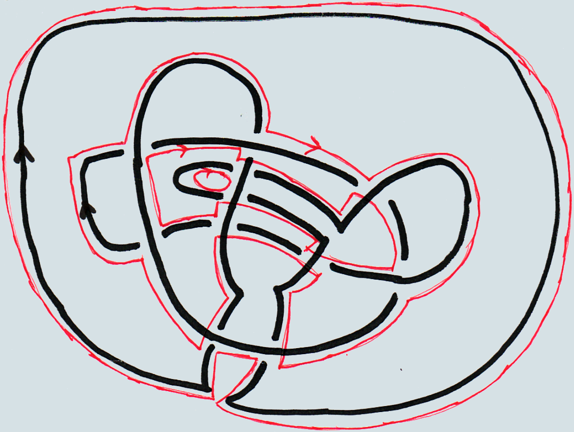
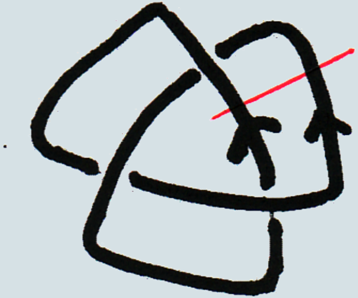


Reading the Braid Word



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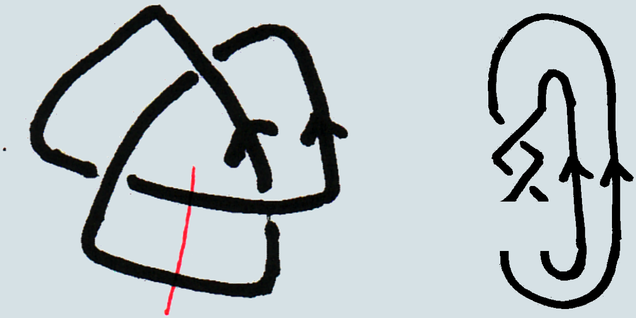

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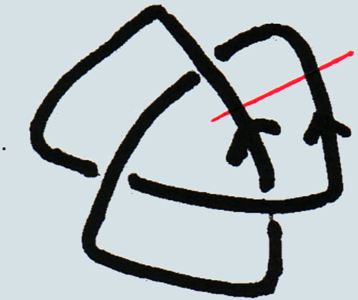
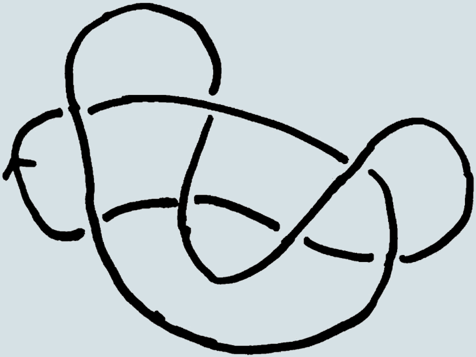
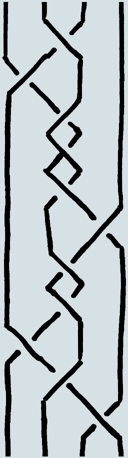


$+1-2+3-1+2-3 \quad \leftrightarrow \quad aaa$

Result





-1, +2, -3, +4, +5, +1, -2, +6, +7, +3,
-4, -7, -6, -5 / - - - - + - + $\leftrightarrow$ BabBBcBBAbC

Conclusion

- Implementation of the Vogel algorithm
- Link between Knotweaver and Seifertview

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- Implementation of the Vogel algorithm
- Link between Knotweaver and Seifertview
- Future options:
  - Smoother connection
  - Multiple components
  - Visualize Vogel algorithm in Knotweaver

Questions?