Knots, Braids, and the Vogel Algorithm

Dan Roozemond

31 May 2007

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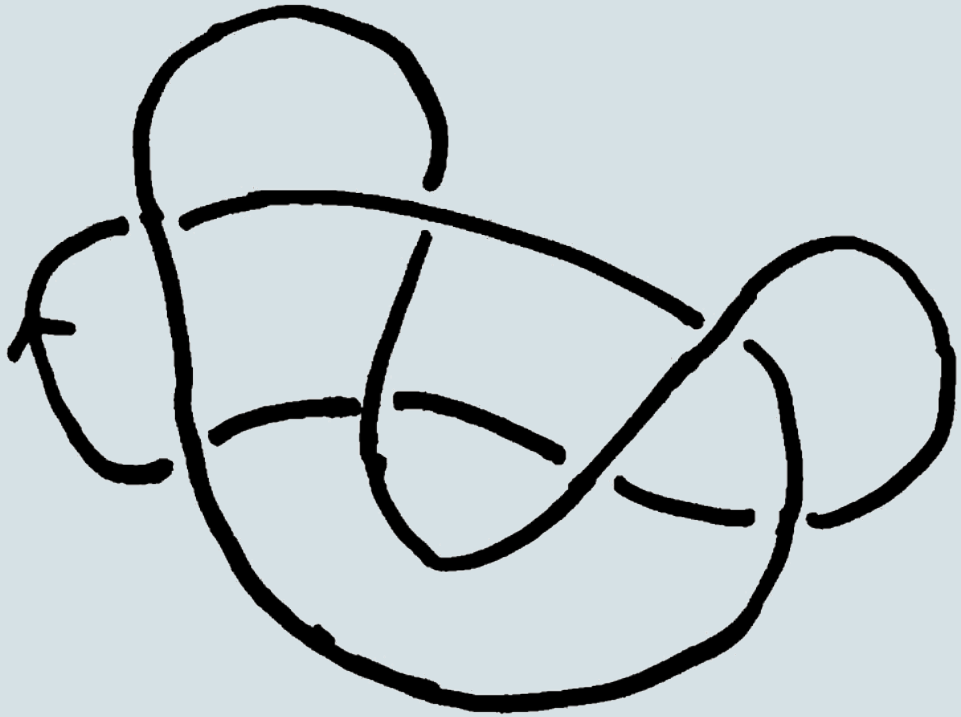
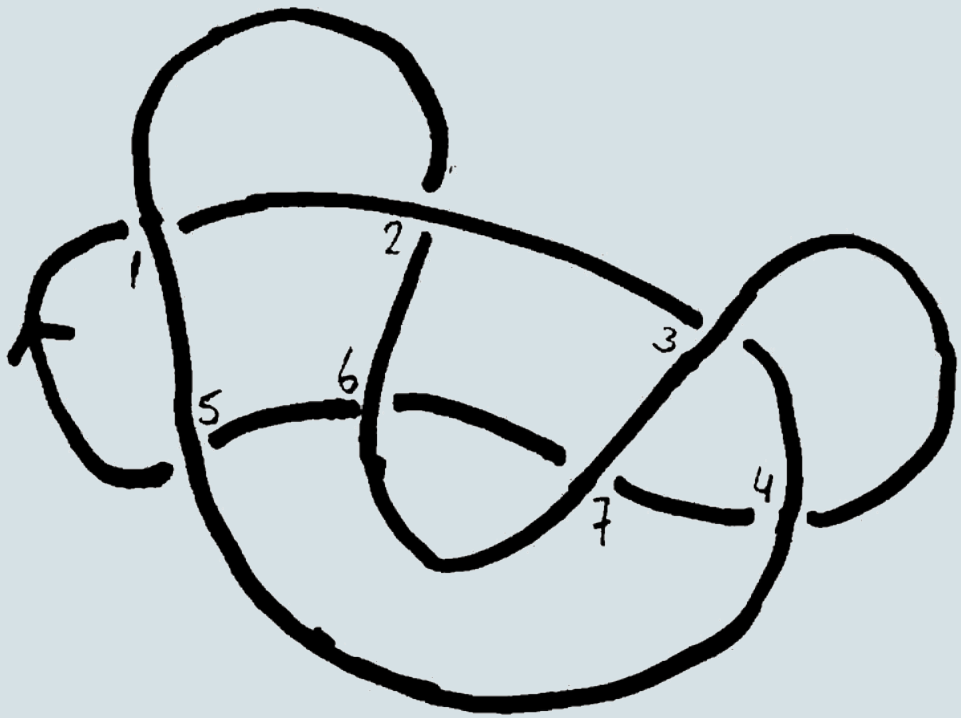
Knots



Representing a Knot: The Gauss Code



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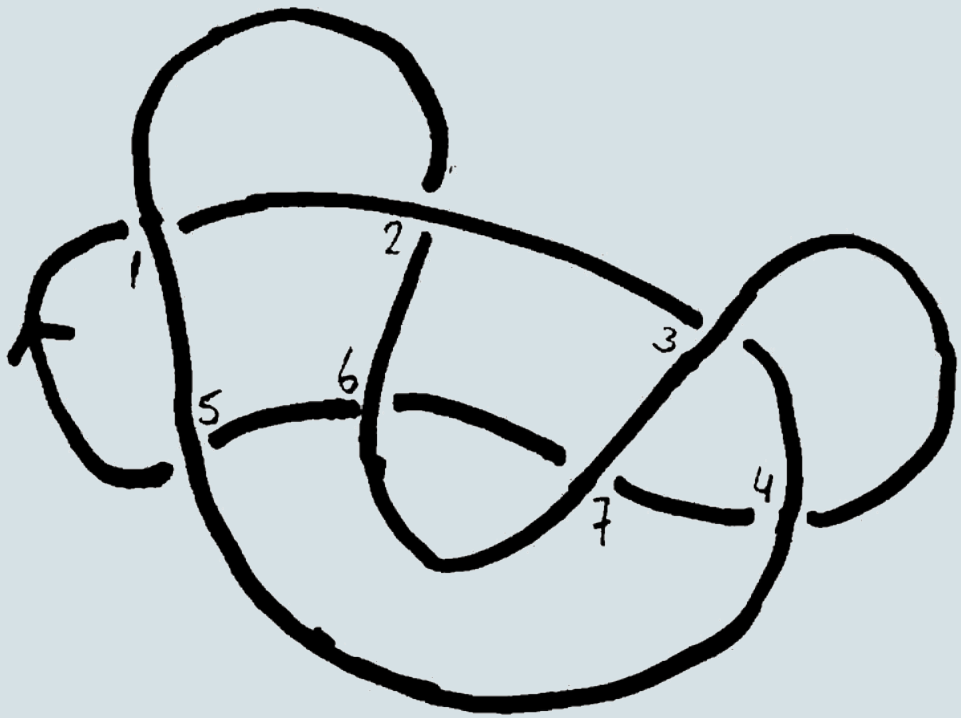
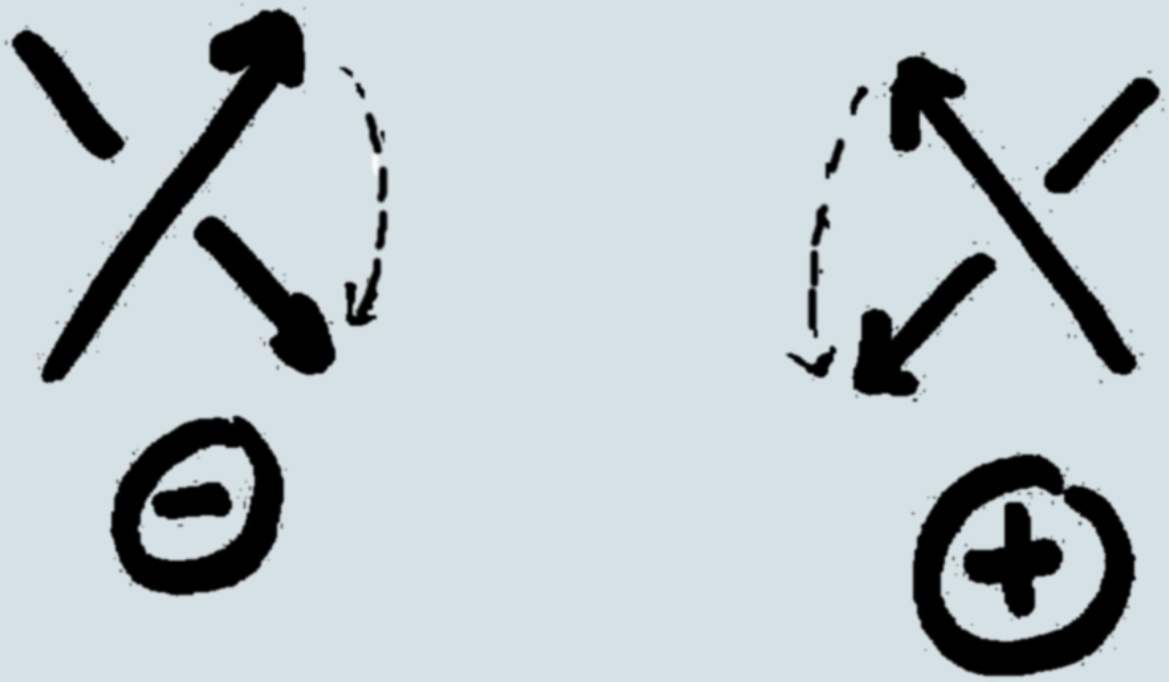


Gauss Code: -1, +2, -3, +4, +5, +1, -2, +6, +7, +3, -4, -7, -6, -5

Representing a Knot: Oriented Gauss Code



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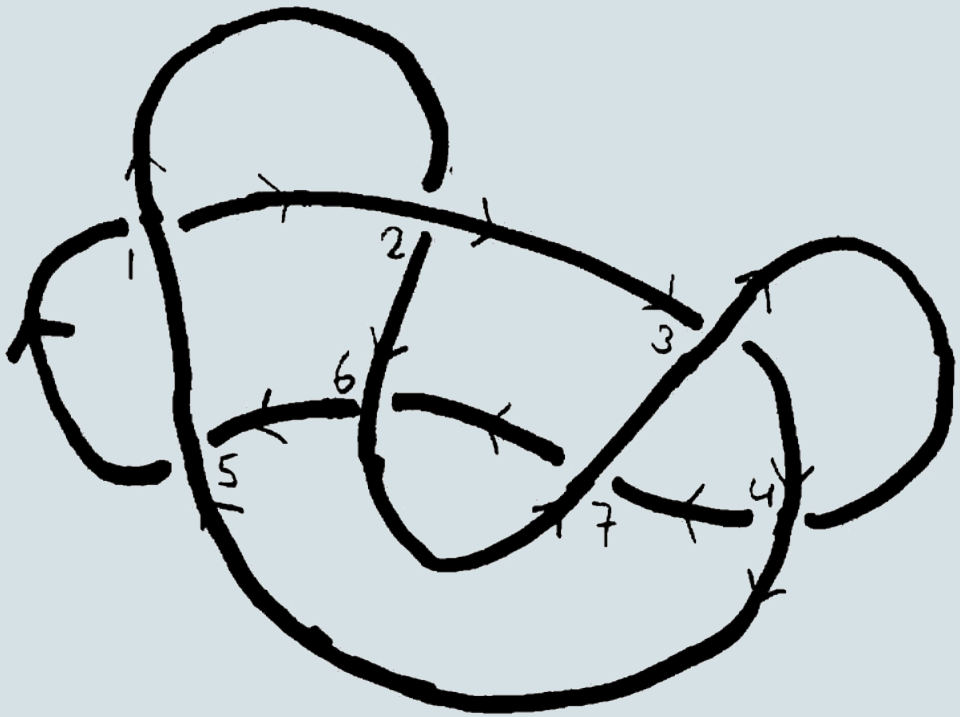

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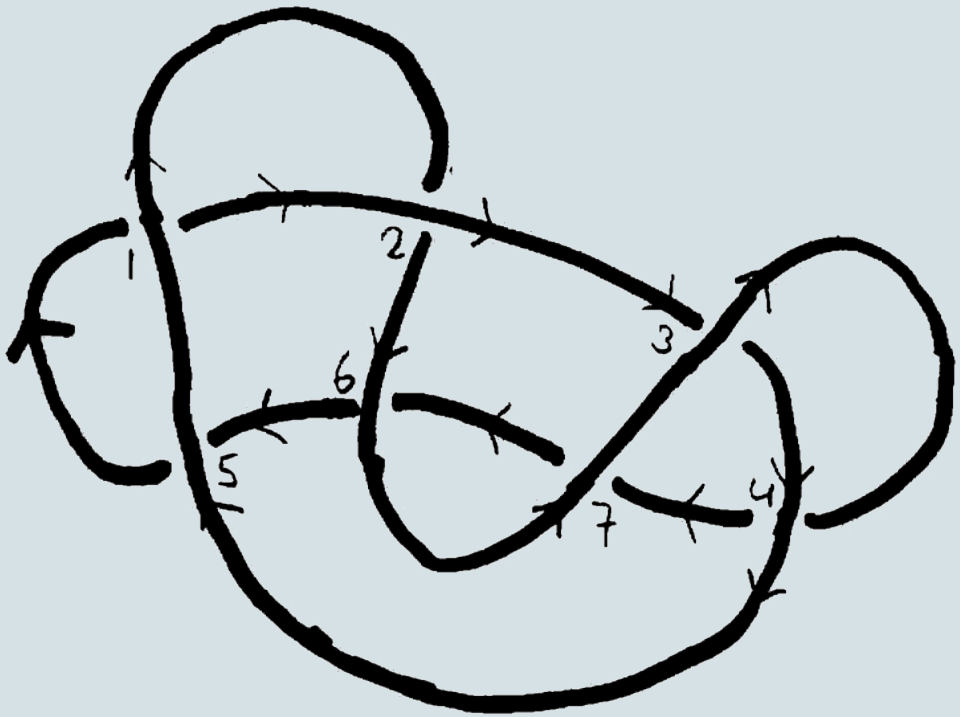
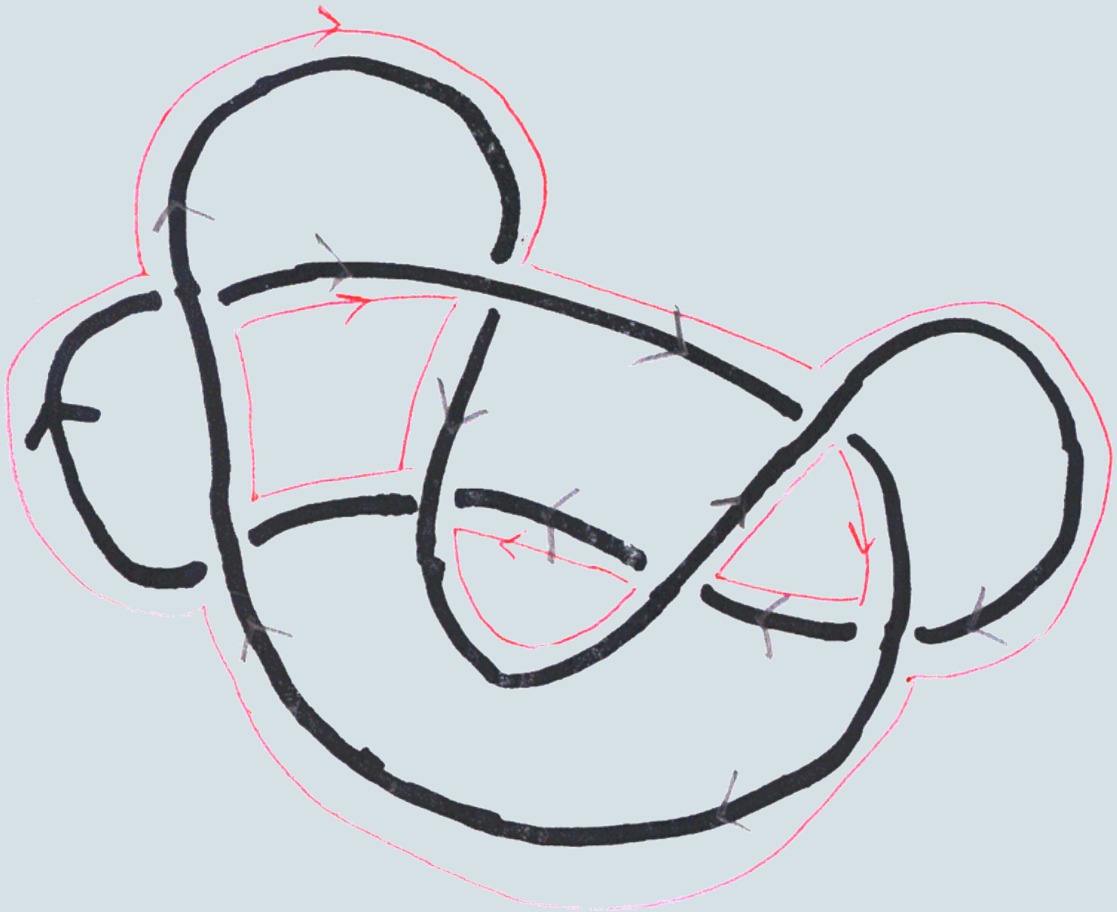


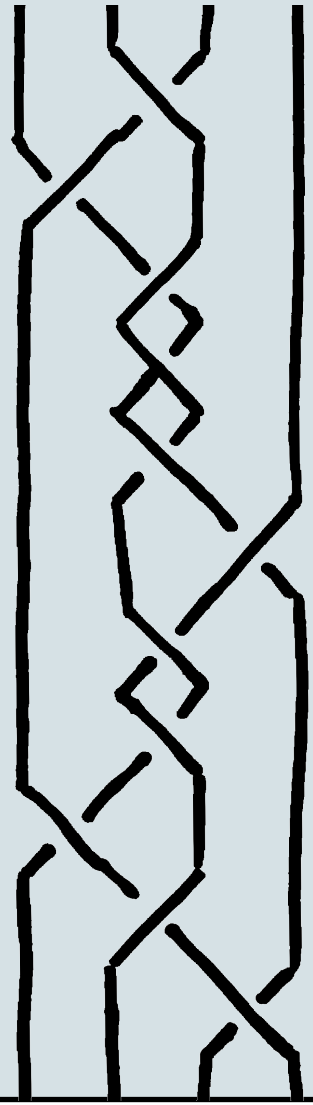
Oriented Gauss Code:

-1, +2, -3, +4, +5, +1, -2, +6, +7, +3, -4, -7, -6, -5 / - - - - + - +

Seifert Circles



Braids



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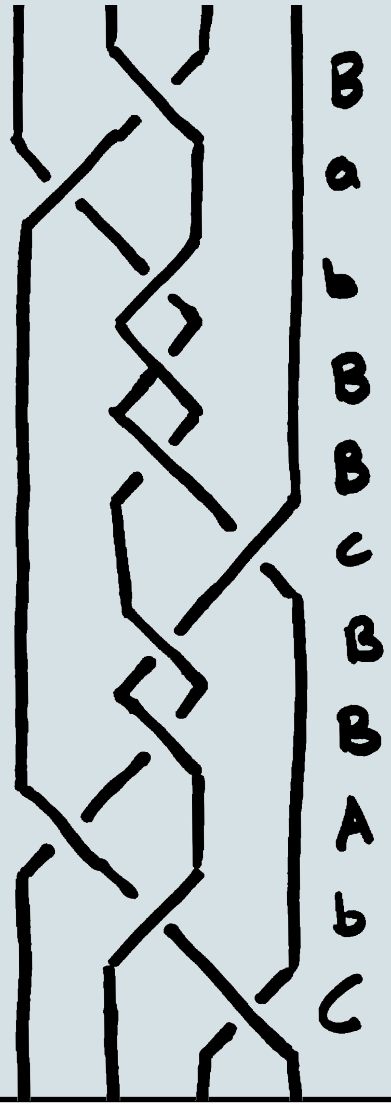
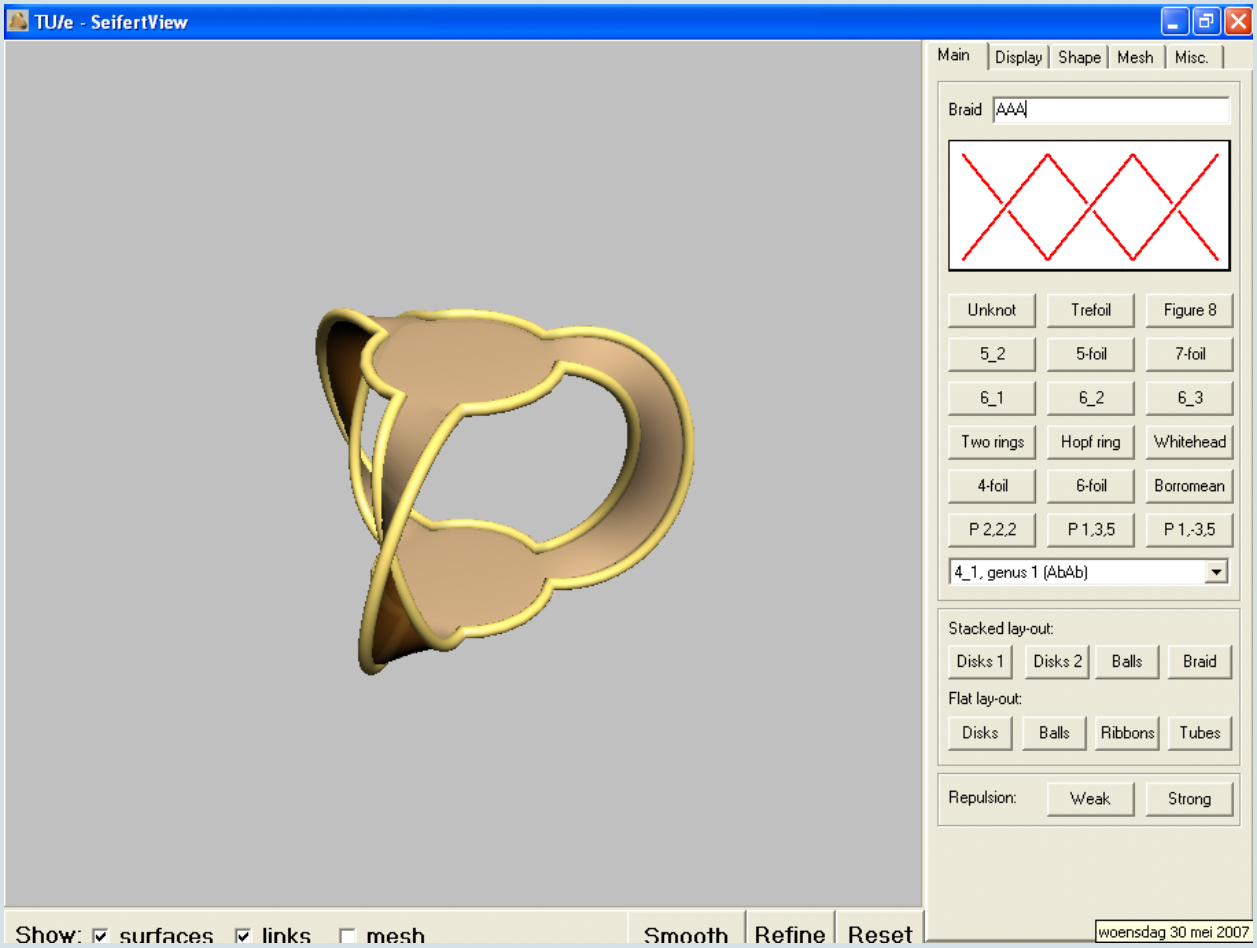


Seifertview

[vanWijk, 2005]



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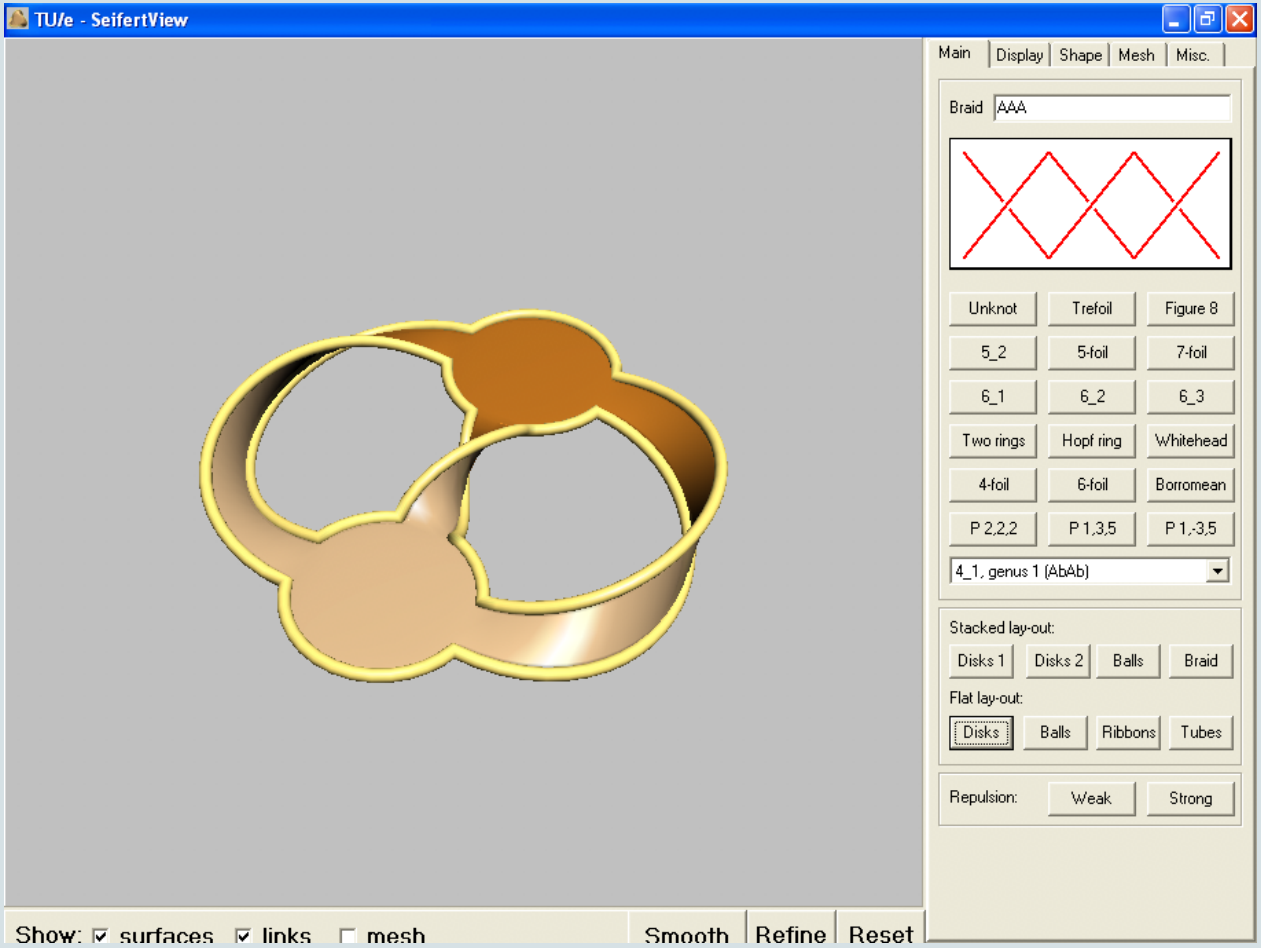

Knots $\leftrightarrow$ Braids

- Seifertview uses braids

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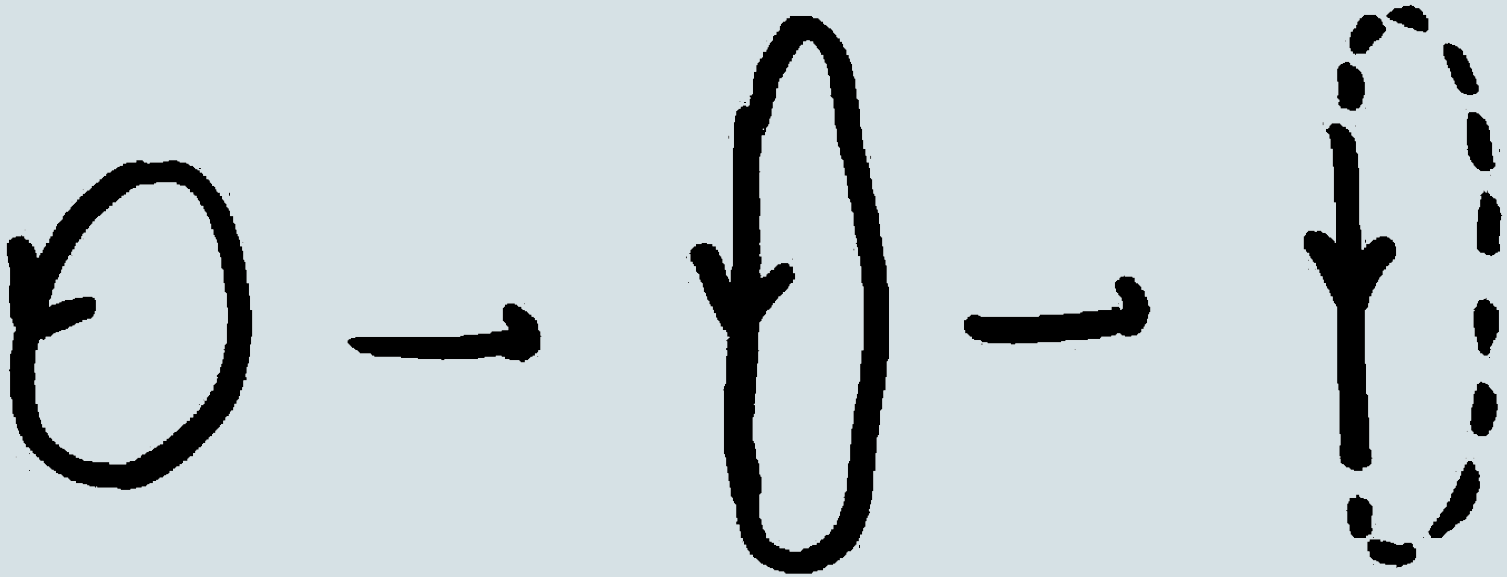
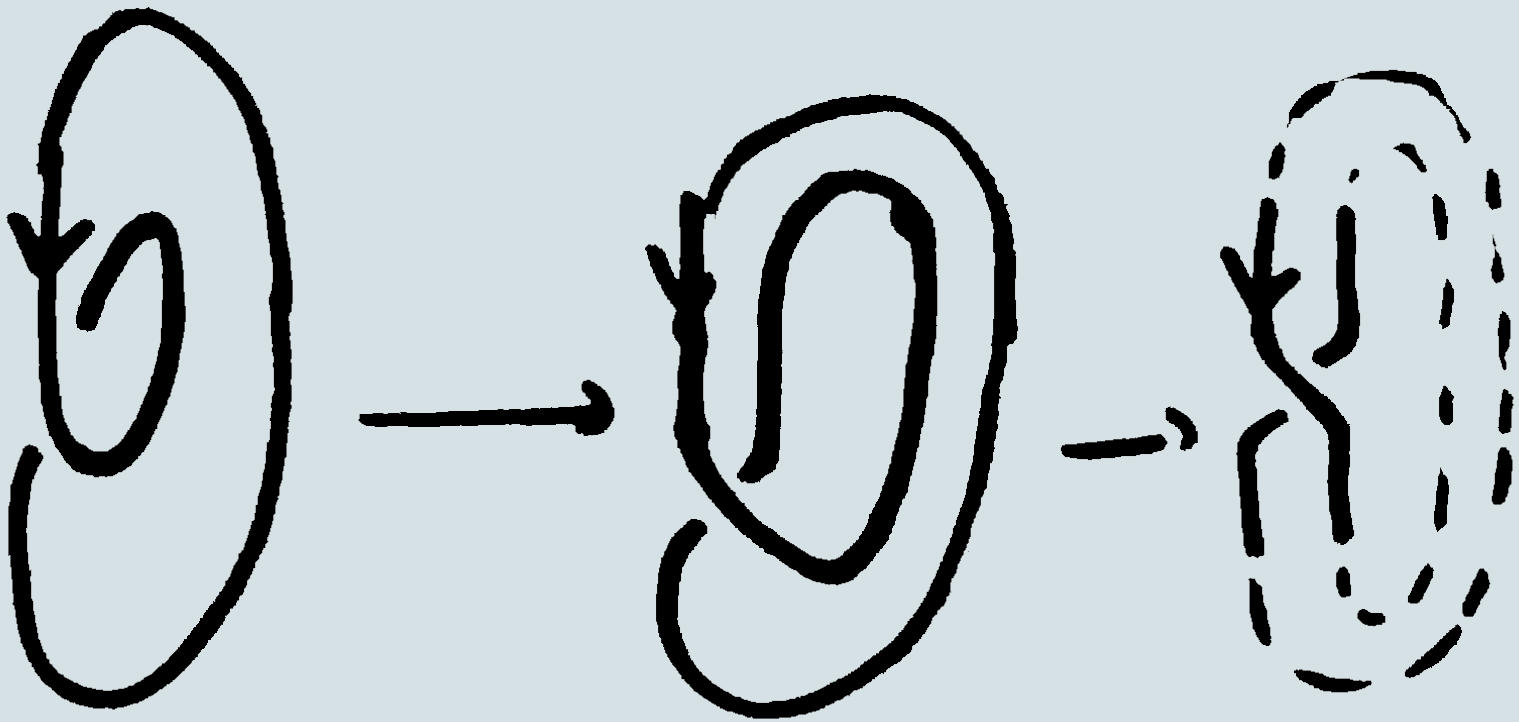
So we want to convert between knots and braids!

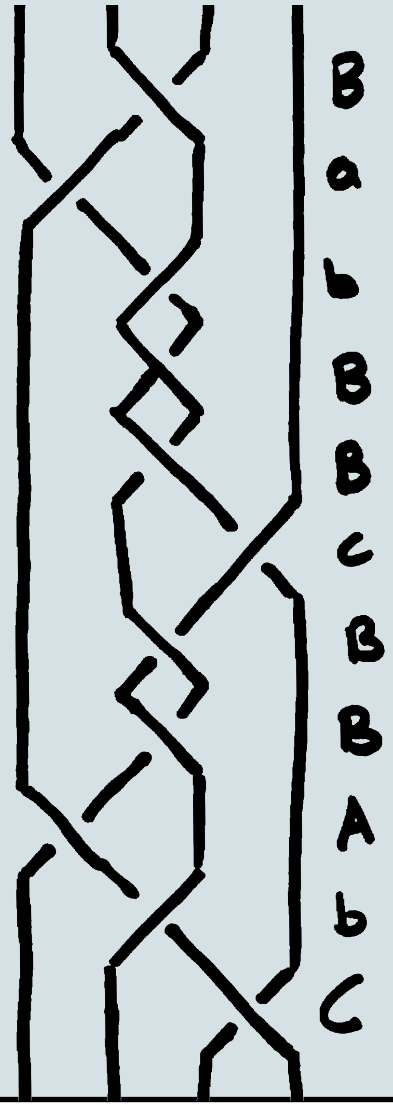
Knots $\leftrightarrow$ Braids



Knots $\leftrightarrow$ Braids



Braids to Knots



Vogel Algorithm

Vogel, 1990

A knot can be brought into suitable form by doing a series of Vogel moves (Reidemeister II moves). The braid word can then easily be read off.

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We have implemented this algorithm in GAP.

Vogel Algorithm: On the Web

The Vogel Algorithm

In *Representation of links by braids, a new algorithm* (Math. Helvetici 65 (1990), 104-113), Vogel describes an algorithm to manipulate a link diagram in such a way that an equivalent braid word may be read from it (Paper via SpringerLink). Andrew Bartholomew describes the algorithm in great detail in one of his papers as well.

We have implemented the algorithm in GAP, and it can be tested from this very website. In case of problems, please contact Dan Roozemond. One word of warning: For the moment, the algorithm only works for knots, i.e. links with only one component.

Enter a Gauss code and crossing signs below:

Gauss code: +1 -2 +3 -1 +2 -3 (e.g. +1 +2 -2 -1)

Signs: +++ (e.g. +-)

Submit Query

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Executing: `vogel([+1,-2,+3,-1,+2,-3],"+++");`
Result: "B2:aaa"

Enter a Gauss code and crossing signs below:

Gauss code: +1 -2 +3 -1 +2 -3 (e.g. +1 +2 -2 -1)

Signs: +++ (e.g. +-)

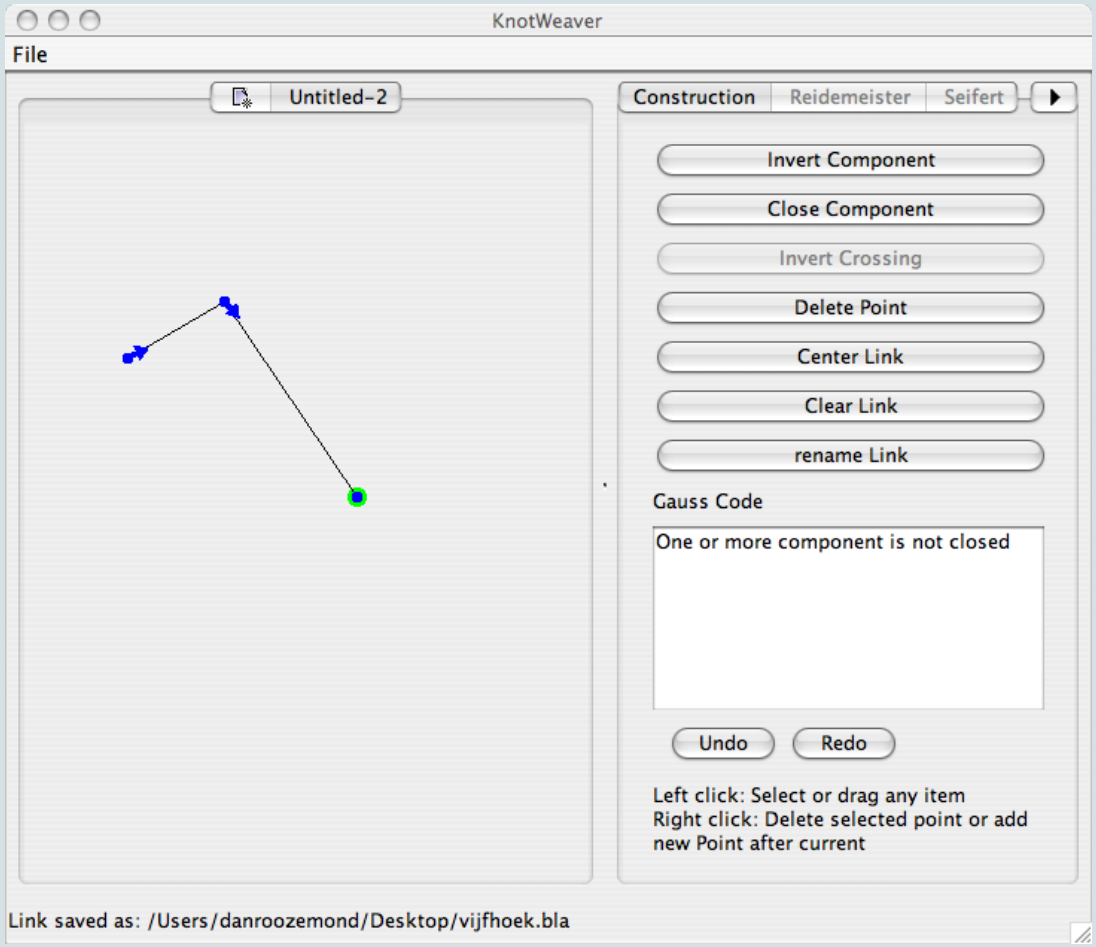
Submit Query

Knots $\leftrightarrow$ Braids

Knot
$\updownarrow$
Oriented Gauss Code
$\updownarrow$
Web page
$\updownarrow$
Braid word
$\updownarrow$
Seifertview

Knotweaver

[Vos, 2007]



Knotweaver

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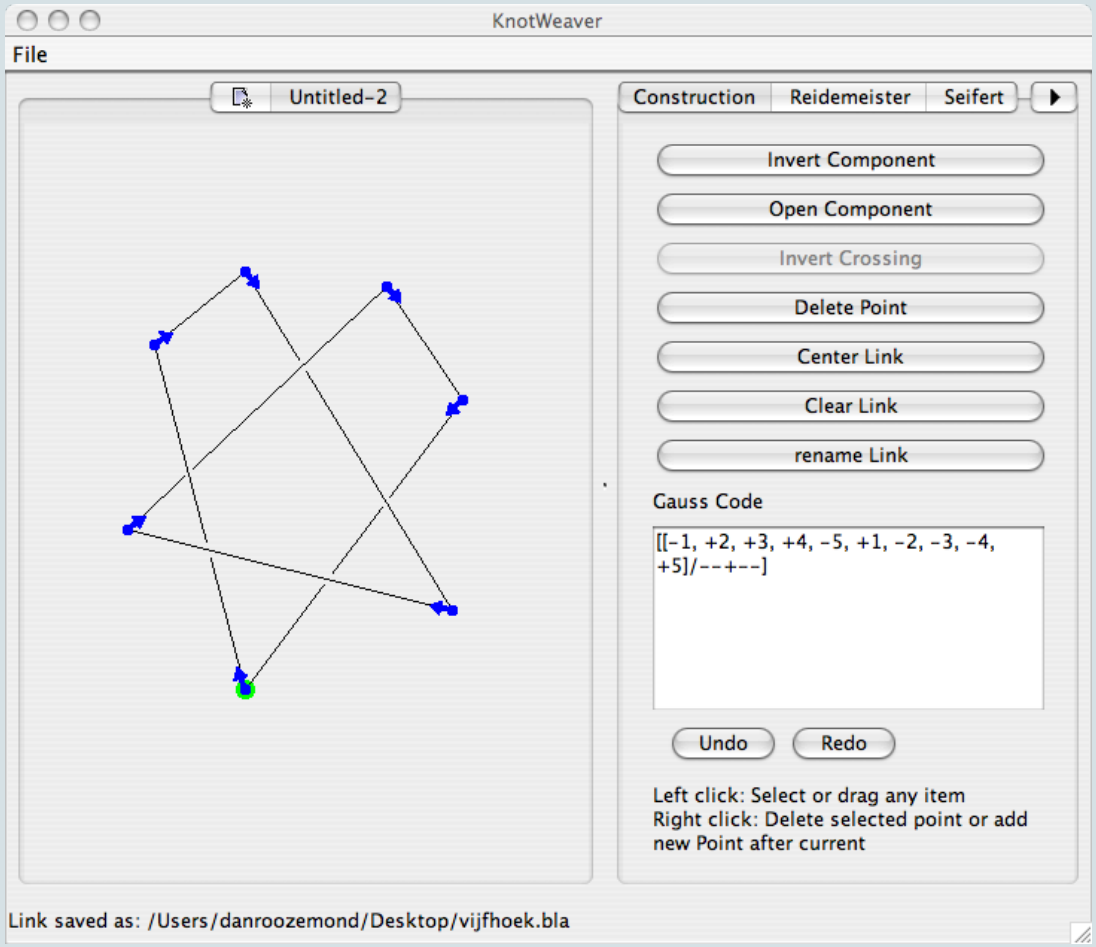

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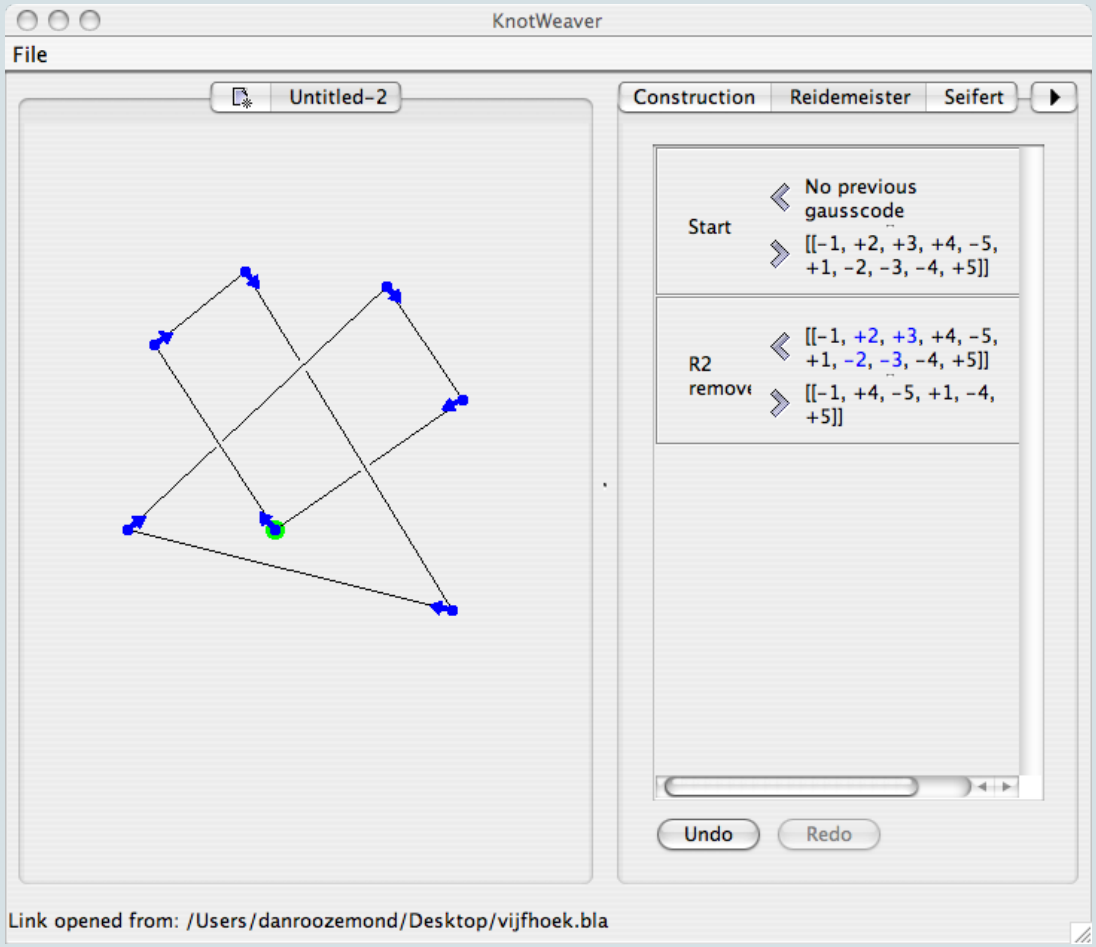
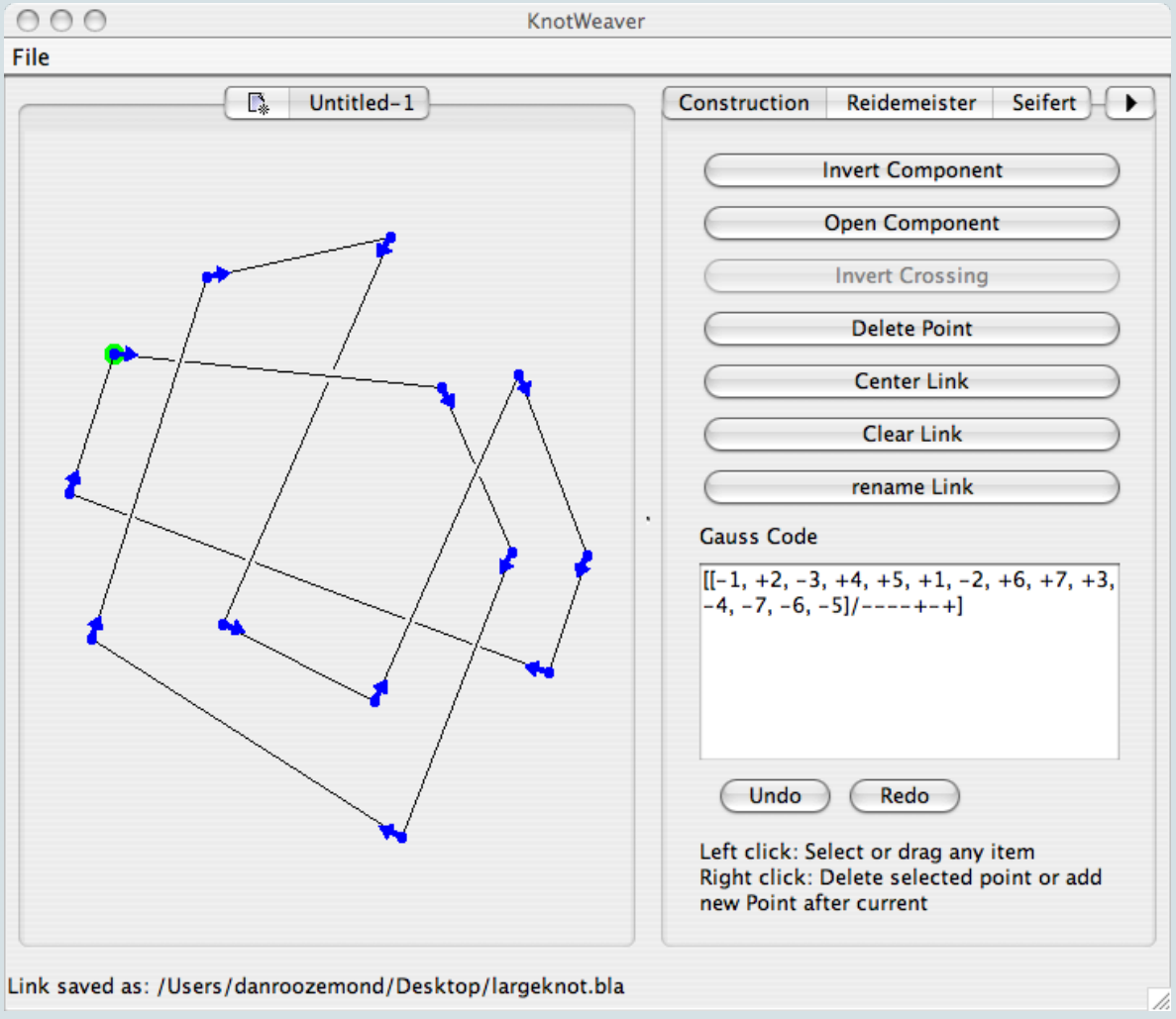


Linking Knotweaver and Seifertview



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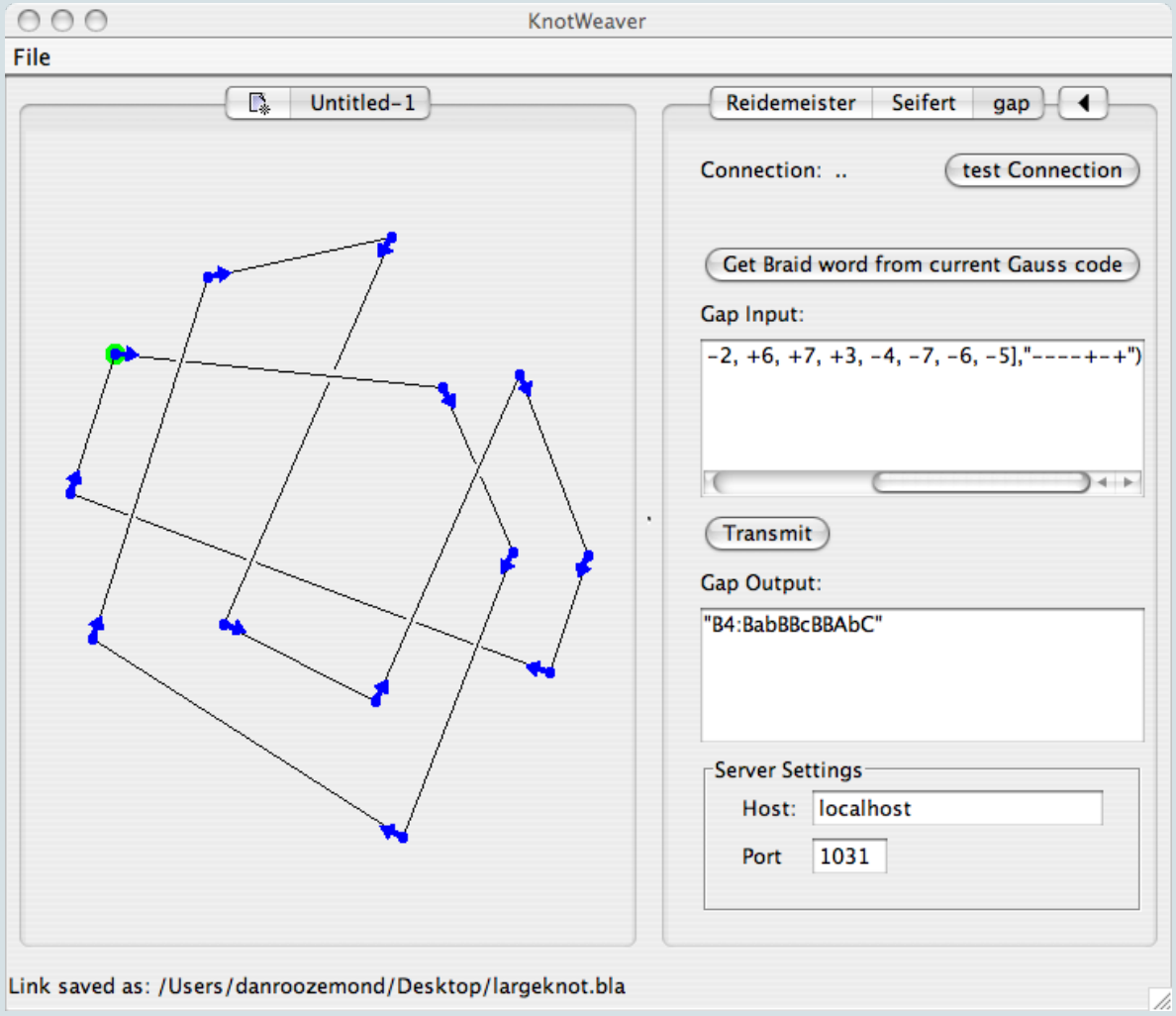

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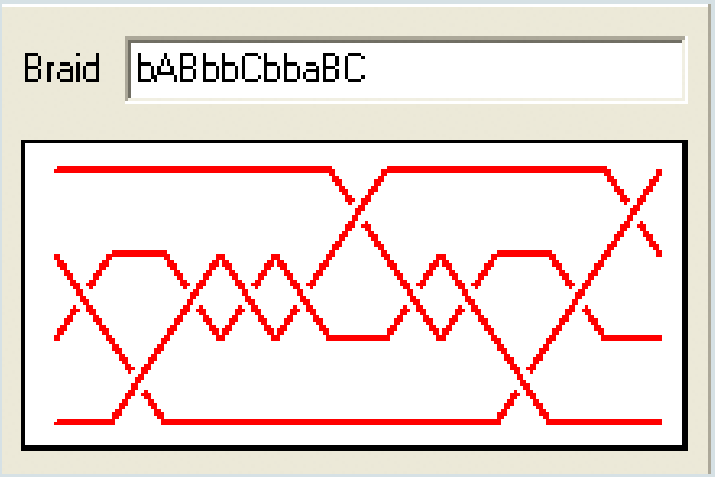

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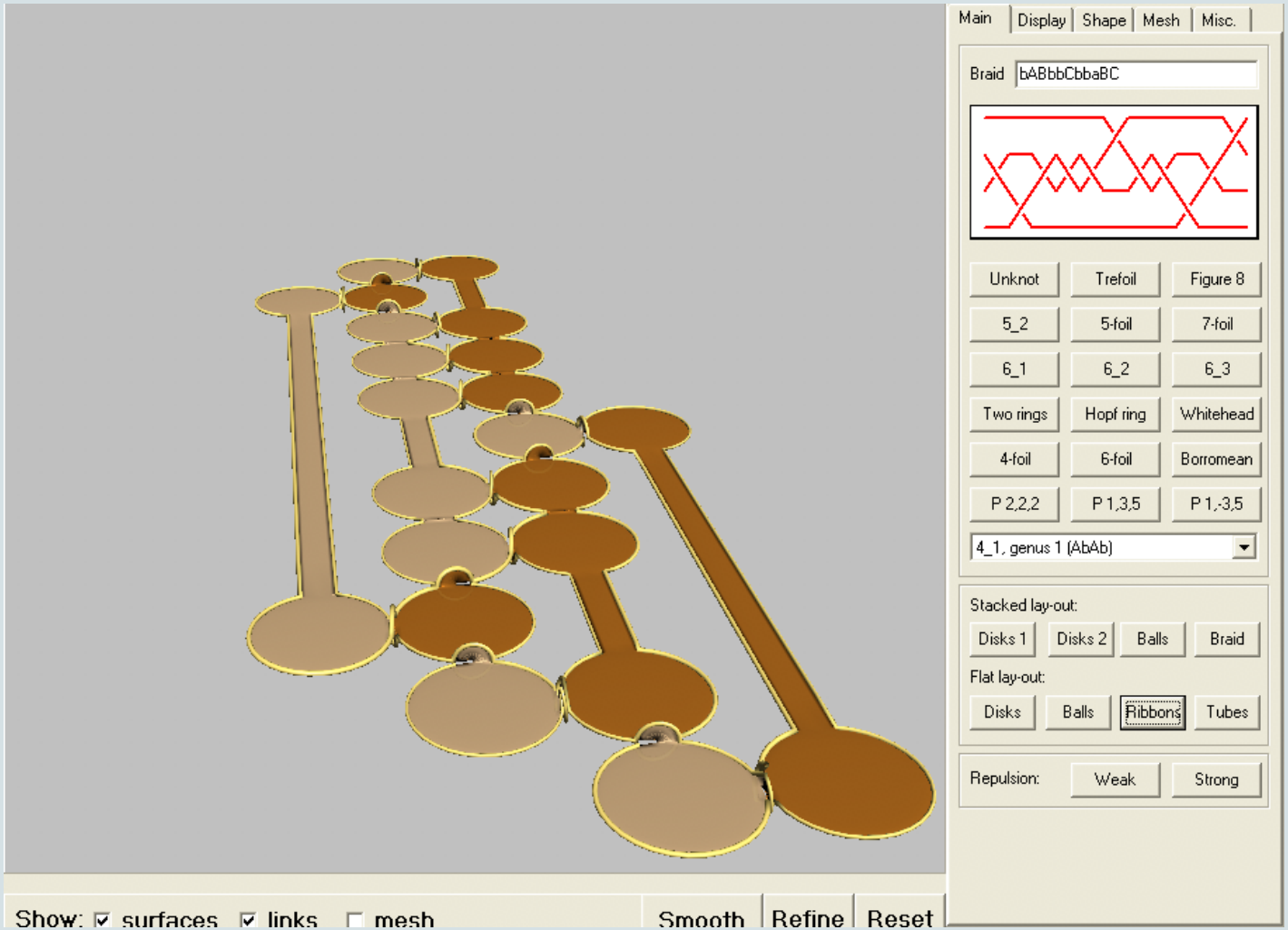
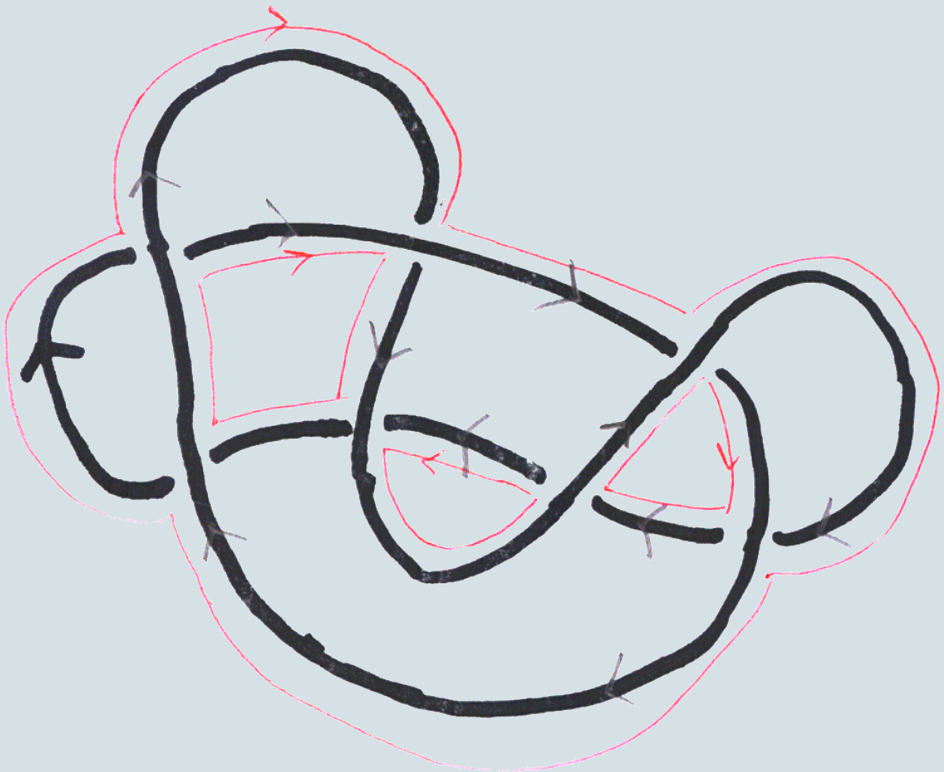


The Vogel Algorithm: A bit more



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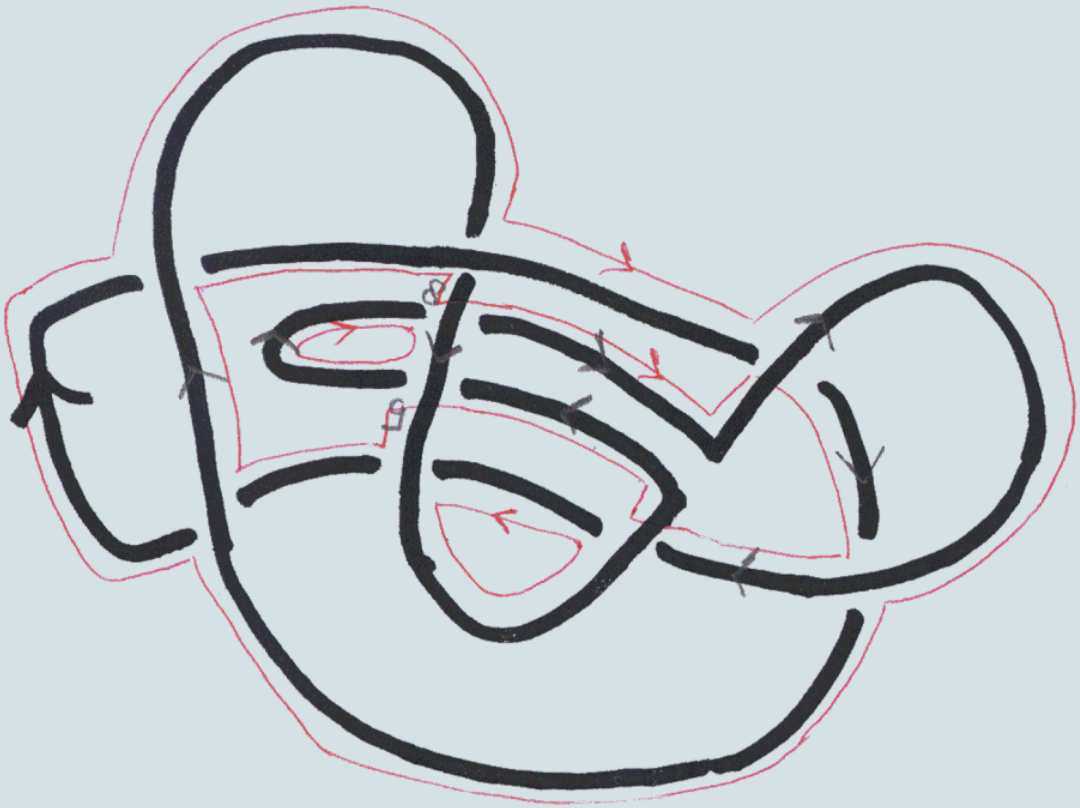

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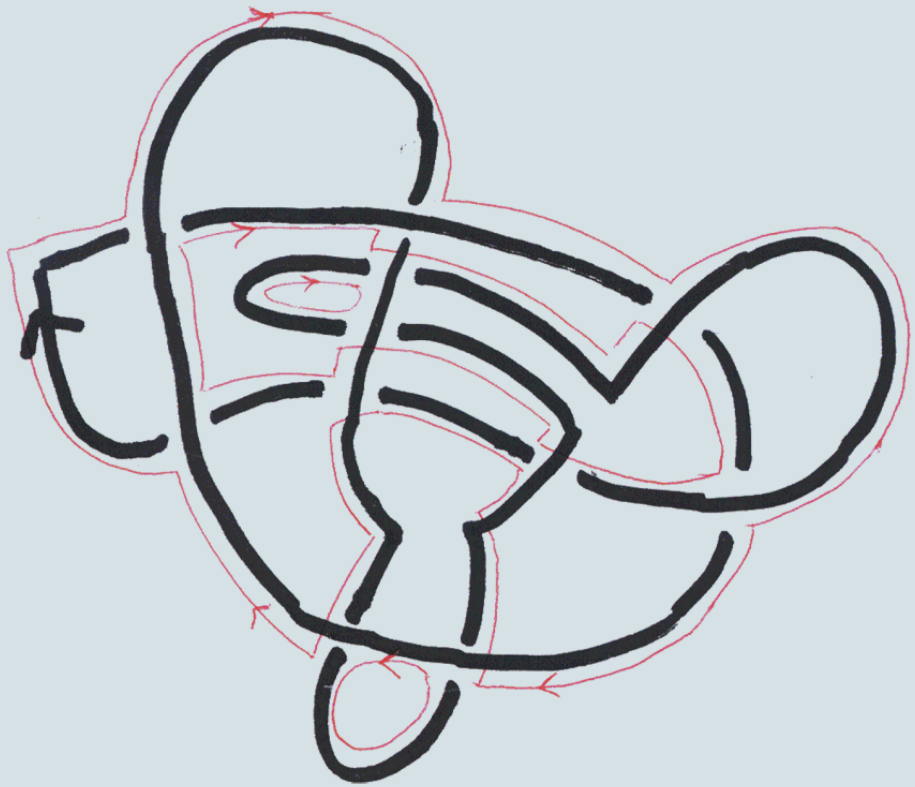

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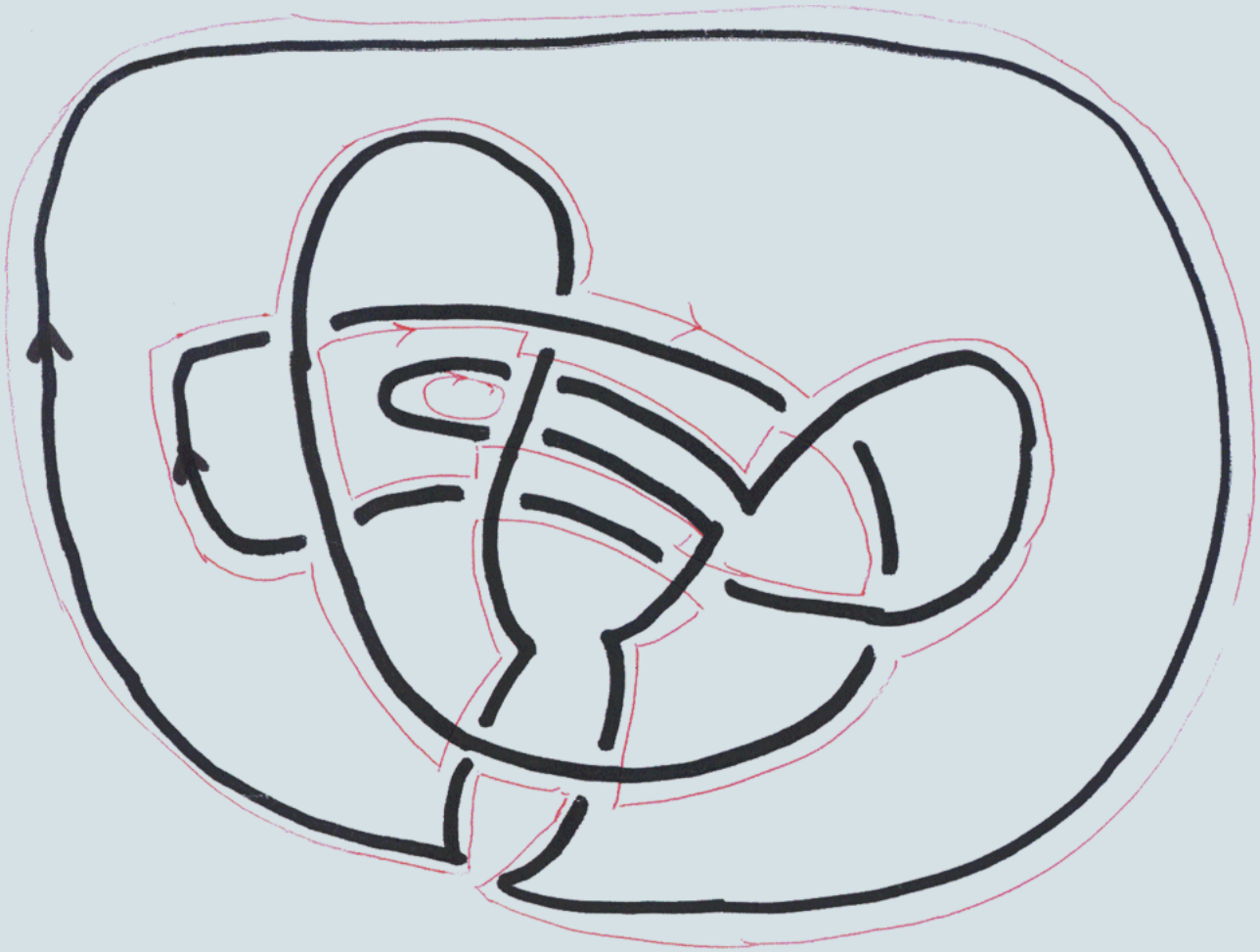

The Vogel Algorithm: A bit more

Conclusion

- Implementation of the Vogel algorithm
- Link between Knotweaver and Seifertview

Conclusion

- Implementation of the Vogel algorithm
- Link between Knotweaver and Seifertview
- Future options:
  - Smoother connection
  - Multiple components
  - Visualize Vogel algorithm in Knotweaver

Questions?