

# Knots, Braids, and the Vogel Algorithm

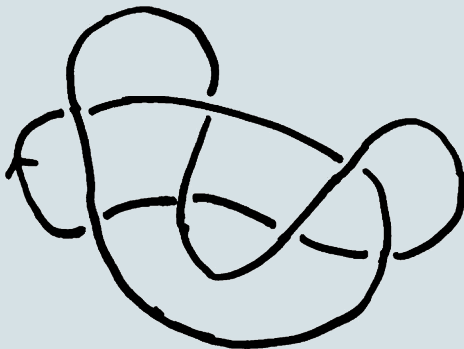
Dan Roozmond

31 May 2007

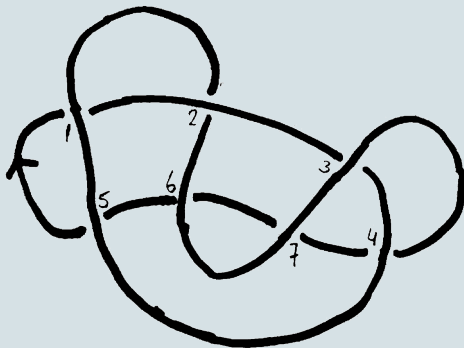
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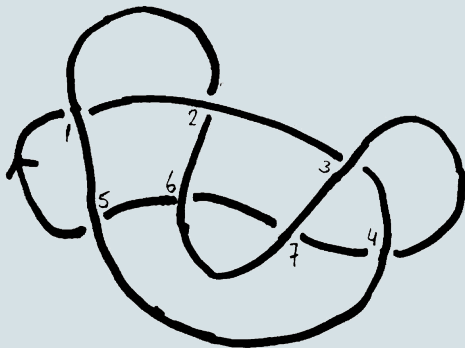
## Knots



## Representing a Knot: The Gauss Code



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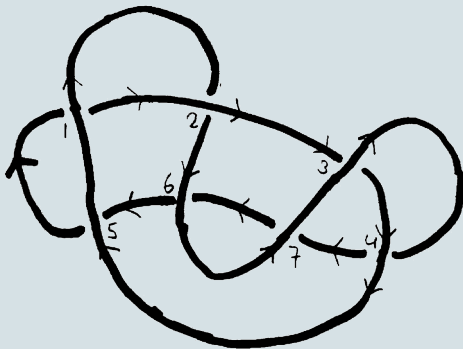


Gauss Code:  $-1, +2, -3, +4, +5, +1, -2, +6, +7, +3, -4, -7, -6, -5$

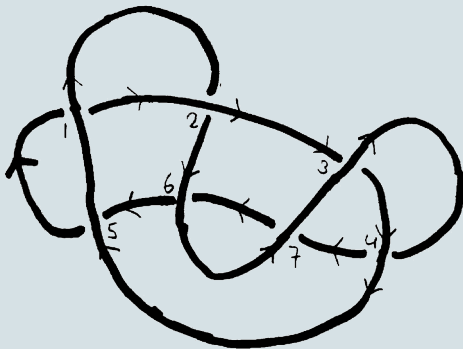
## Representing a Knot: Oriented Gauss Code



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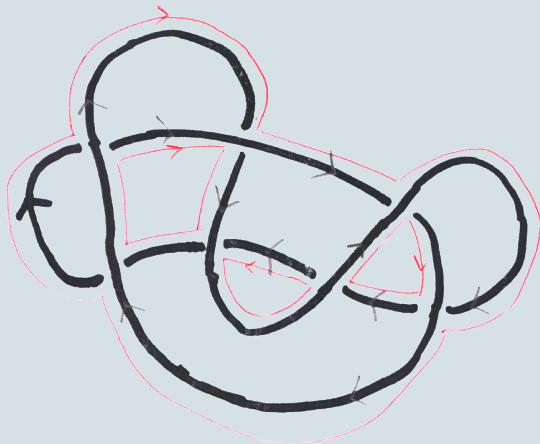


Oriented Gauss Code:

$-1, +2, -3, +4, +5, +1, -2, +6, +7, +3, -4, -7, -6, -5 / - - - - + - +$



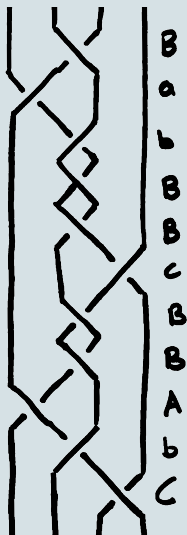
## Seifert Circles



## Braids

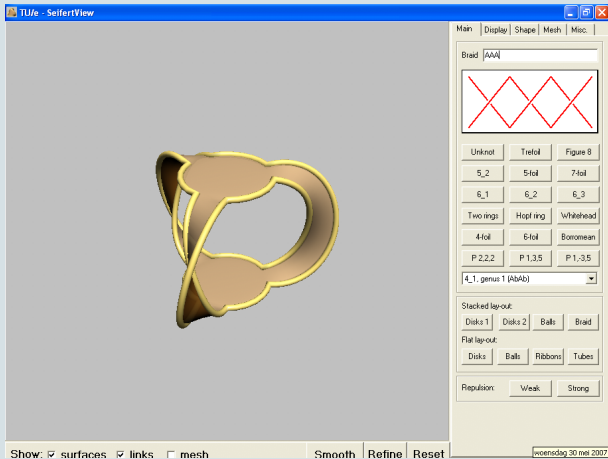


## Braids



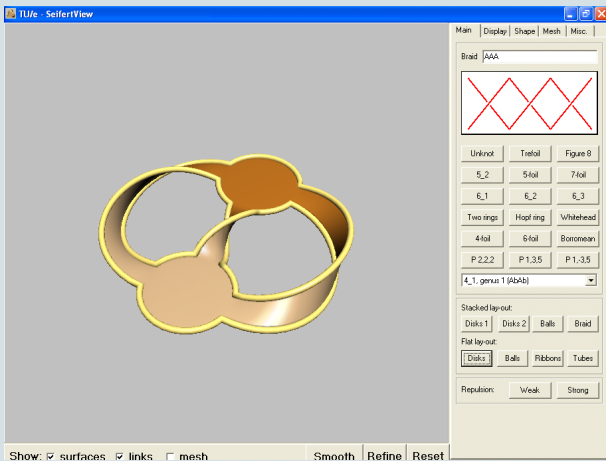
## Seifertview

[vanWijk, 2005]



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[vanWijk, 2005]



# Knots $\leftrightarrow$ Braids

- ▶ Seifertview uses braids

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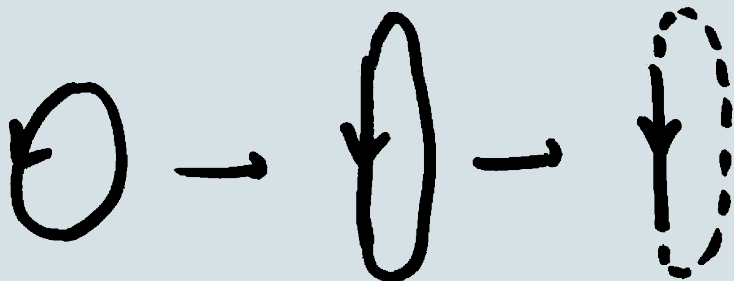
- ▶ Seifertview uses braids
- ▶ Knots and braids are essentially the same things!
- ▶ Braidgroups, Artin groups, Hecke algebras, etc.

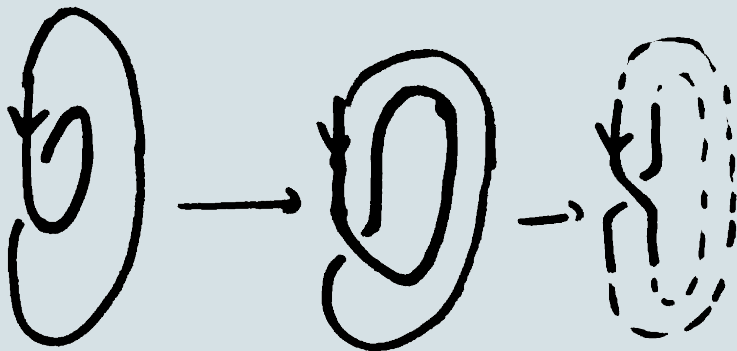


# Knots $\leftrightarrow$ Braids

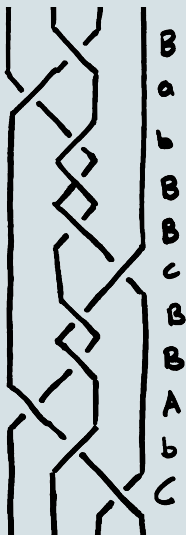
- ▶ Seifertview uses braids
- ▶ Knots and braids are essentially the same things!
- ▶ Braidgroups, Artin groups, Hecke algebras, etc.

So we want to convert between knots and braids!

Knots  $\leftrightarrow$  Braids

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## Braids to Knots



# Vogel Algorithm

## Vogel, 1990

A knot can be brought into suitable form by doing a series of Vogel moves (Reidemeister II moves). The braid word can then easily be read off.

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We have implemented this algorithm in GAP.

## Vogel Algorithm: On the Web

### The Vogel Algorithm

In *Representation of links by braids, a new algorithm* (Math. Helvetici 65 (1990), 104-113), Vogel describes an algorithm to manipulate a link diagram in such a way that an equivalent braid word may be read from it ([Paper via SpringerLink](#)). Andrew Bartholomew describes the algorithm in great detail in [one of his papers](#) as well.

We have implemented the algorithm in [GAP](#), and it can be tested from this very website. In case of problems, please contact [Dan Roozmond](#). One word of warning: For the moment, the algorithm only works for knots, i.e. links with only one component.

Enter a Gauss code and crossing signs below:

**Gauss code:**  (e.g. +1 +2 -2 -1 )

**Signs:**  (e.g. +- )

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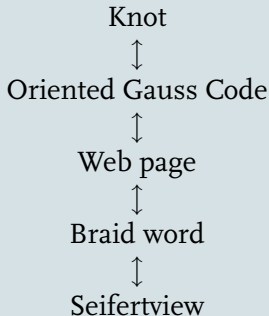
```
Executing: vogel1([+1,-2,+3,-1,+2,-3], "+++");  
Result: "B2:aaa"
```

Enter a Gauss code and crossing signs below:

**Gauss code:**  (e.g. +1 +2 -2 -1 )

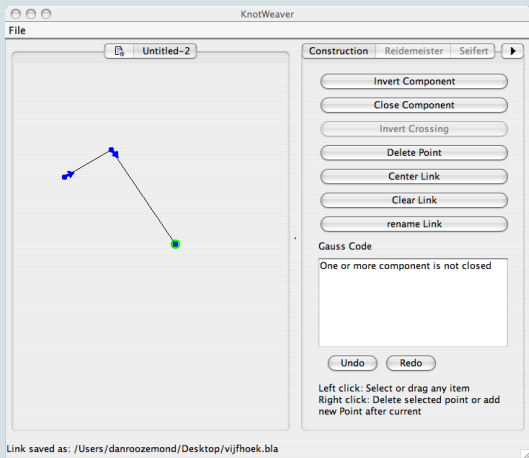
**Signs:**  (e.g. +- )



Knots  $\leftrightarrow$  Braids

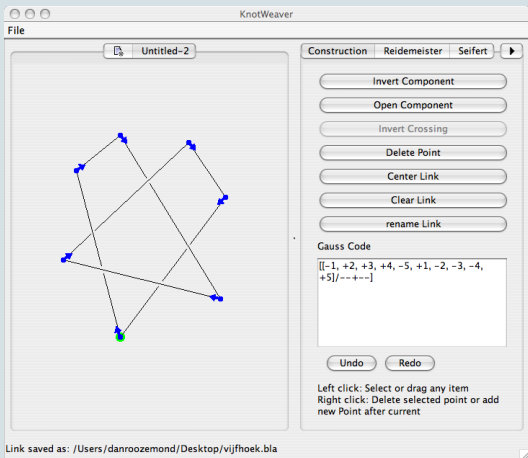
## Knotweaver

[Vos, 2007]



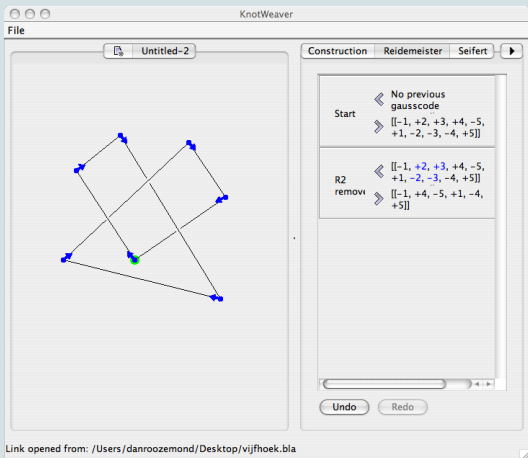
## Knotweaver

[Vos, 2007]

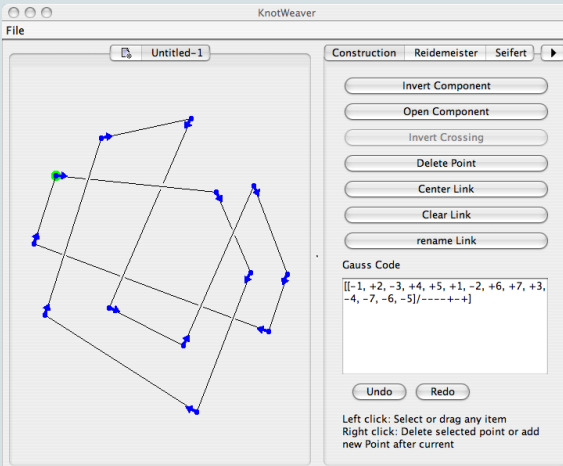


## Knotweaver

[Vos, 2007]



## Linking Knotweaver and Seifertview



The screenshot shows the KnotWeaver application window. The main canvas displays a complex knot diagram with several blue points and lines. A toolbar on the right contains buttons for various operations: Invert Component, Open Component, Invert Crossing, Delete Point, Center Link, Clear Link, and rename Link. Below the toolbar is a text area for Gauss Code, containing the code:  $[[-1, +2, -3, +4, +5, +1, -2, +6, +7, +3, -4, -7, -6, -5]/-----+]$ . At the bottom of the toolbar are Undo and Redo buttons. A legend at the bottom right explains the click actions: Left click: Select or drag any item; Right click: Delete selected point or add new Point after current.

File

Untitled-1

Construction Reidemeister Seifert

Invert Component

Open Component

Invert Crossing

Delete Point

Center Link

Clear Link

rename Link

Gauss Code

```
[[-1, +2, -3, +4, +5, +1, -2, +6, +7, +3, -4, -7, -6, -5]/-----+]
```

Undo Redo

Left click: Select or drag any item  
Right click: Delete selected point or add new Point after current

Link saved as: /Users/danroozemond/Desktop/largeknot.bla

## Linking Knotweaver and Seifertview

KnotWeaver

File

Untitled-1

Reidemeister Seifert gap

Connection: .. test Connection

Get Braid word from current Gauss code

Gap Input:

```
-2, +6, +7, +3, -4, -7, -6, -5],\"-----++*)
```

Transmit

Gap Output:

```
\"B4:BabBbcBBAbC\"
```

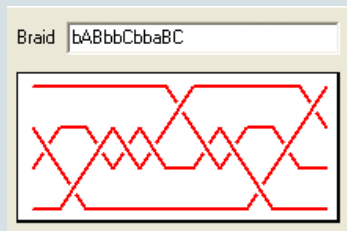
Server Settings

Host: localhost

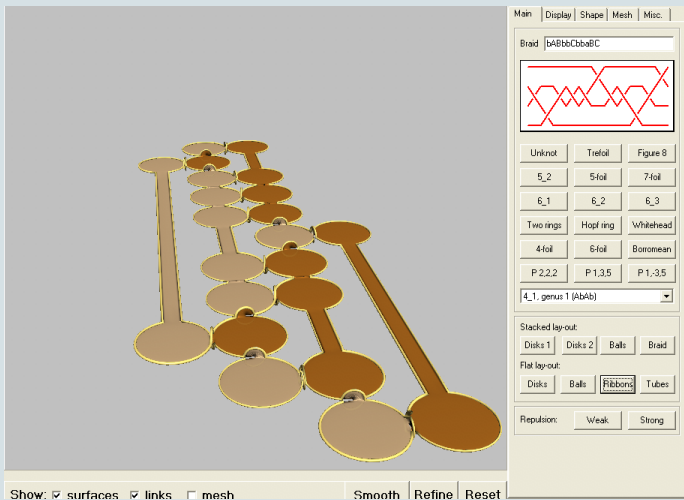
Port: 1031

Link saved as: /Users/danroozemond/Desktop/largeknot.bla

# Linking Knotweaver and Seifertview

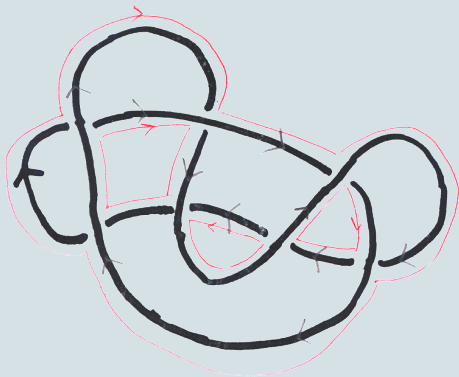


## Linking Knotweaver and Seifertview

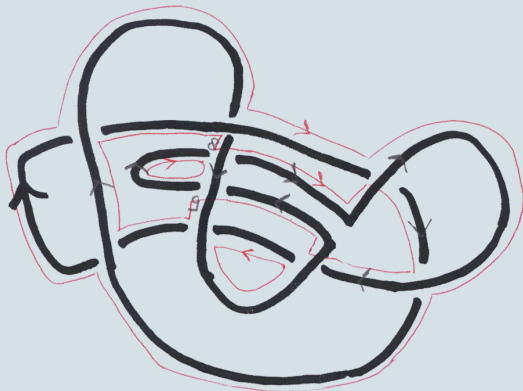




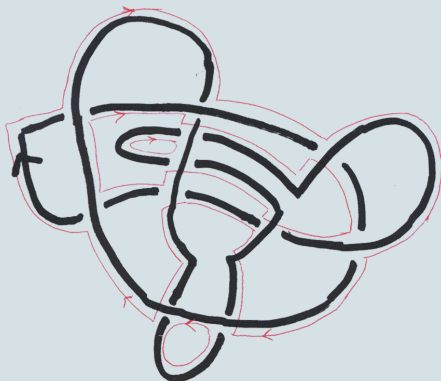
# The Vogel Algorithm: A bit more



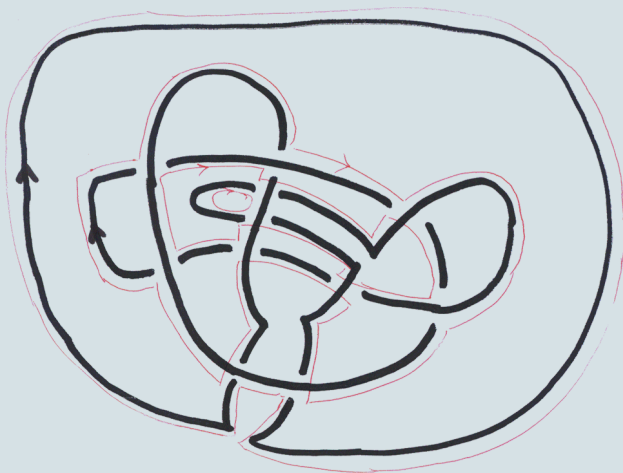
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## Conclusion

- ▶ Implementation of the Vogel algorithm
- ▶ Link between Knotweaver and Seifertview

# Conclusion

- ▶ Implementation of the Vogel algorithm
- ▶ Link between Knotweaver and Seifertview
- ▶ Future options:
  - ▶ Smoother connection
  - ▶ Multiple components
  - ▶ Visualize Vogel algorithm in Knotweaver

# Questions?